

## Math 308 E Test Prep Problems - Week #2

- (1) Matt is a software engineer writing a script involving 6 tasks. Each must be done one after the other. Let  $t_i$  be the time for the  $i$ th task. These times have a certain structure:
- Any 3 adjacent tasks will take half as long as the next two tasks.
  - The second task takes 1 second.
  - The fourth task takes 10 seconds.
- (a) Write an augmented matrix for the system of equations describing the length of each task.
- (b) Reduce this augmented matrix to reduced echelon form.
- (c) Suppose he knows additionally that the sixth task takes 20 seconds and the first three tasks will run in 50 seconds. Write the extra rows that you would add to your answer in (b) to take account of this new information.
- (d) Solve the system of equations in (c).

### Solution by Groups A5, B5, C5 – due in class on Wednesday 1/10

- (2) (after 2.1) When Jake works from  $\vec{h}$ ome, he typically spends 40 minutes of each hour on research, and 10 on teaching, and drinks half a cup of coffee. (The remaining time is spent on the internet.) For each hour he works in the math  $\vec{d}$ epartment, he spends around 20 minutes on research and 30 on teaching, and doesn't drink any coffee. Lastly, if he works at a  $\vec{c}$ offeeshop for an hour, he spends 25 minutes each on research and teaching, and drinks a cup of coffee.
- (Note: be careful about units of minutes versus hours.)

(a) Last week, Jake spent 10 hours working from home, 15 hours working in his office in Padelford Hall, and 2 hours working at Cafe Allegro. Compute what was accomplished, and express the result as a vector equation.

(b) This week, Jake has 15 hours of research to work on and 10 hours of work related to teaching. He also wants 11 cups of coffee, because... of... very important reasons. How much time should he spend working from home, from his office, and from the coffeeshop?

(c) Describe the situation in part (b) as a vector equation and a matrix equation  $A\vec{t} = \vec{w}$ . What do the vectors  $\vec{t}$  and  $\vec{w}$  mean in this context? For which other vectors  $\vec{w}$  does the equation  $A\vec{t} = \vec{w}$  have a solution?

(d) Jake tries working in the math department  $\vec{l}$ ounge for an hour, and gets 30 minutes of research and 20 minutes of teaching work done, while having time to drink  $\frac{1}{3}$  of a cup of coffee. Not bad. But Jake's colleague Vasu claims that there's no need to work in the lounge – the other options already give enough flexibility. Is he right? Explain mathematically.

### Solution by Groups A6, B6, C6 – due in class on Wednesday 1/10

- (3) (after 2.1) Find a  $3 \times 4$  matrix  $A$ , in *reduced* echelon form, with free variable  $x_3$ , such that the general solution of the equation  $A\mathbf{x} = \begin{bmatrix} -1 \\ 1 \\ 6 \end{bmatrix}$  is

$$\mathbf{x} = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 6 \end{bmatrix} + s \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix},$$

where  $s$  is any real number.

**Solution by Groups A7, B7, C7 – due in class on Friday 1/12**

- (4) (after 2.2) Find all values  $z_1$  and  $z_2$  such that  $(2, -1, 3)$ ,  $(1, 2, 2)$ , and  $(-4, z_1, z_2)$  do not span  $\mathbb{R}^3$ .

**Solution by Groups A8, B8, C8 – due in class on Friday 1/12**

- (5) (after 2.3) (a) Let  $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ ,  $\mathbf{a}_2 = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$ , and  $\mathbf{a}_3 = \begin{bmatrix} t \\ -3 \\ -7 \end{bmatrix}$ . Find all values of  $t$  for which there will be a unique solution to  $\mathbf{a}_1x_1 + \mathbf{a}_2x_2 + \mathbf{a}_3x_3 = \mathbf{b}$  for every vector  $\mathbf{b}$  in  $\mathbb{R}^3$ . Explain your answer.  
 (b) Are the vectors  $\mathbf{a}_1$  and  $\mathbf{a}_2$  from part (a) linearly independent? Explain your answer.

- (c) Let  $\mathbf{a}_1$ ,  $\mathbf{a}_2$  and  $\mathbf{a}_3$  be as in (a). Let  $\mathbf{a}_4 = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}$ . Without doing any further calculations, find all values of  $t$  for which there will be a unique solutions to  $\mathbf{a}_1y_1 + \mathbf{a}_2y_2 + \mathbf{a}_3y_3 + \mathbf{a}_4y_4 = \mathbf{c}$  for every vector  $\mathbf{c}$  in  $\mathbb{R}^3$ . Explain your answer.

**Solution by Groups A9, B9, C9 – due in class on Wednesday 1/17**

- (6) For each of the situations described below, **give an example** (if it's possible) or **explain why it's not possible**.  
 (a) A set of vectors that does not span  $\mathbb{R}^3$ . After adding one more vector, the set does span  $\mathbb{R}^3$ .  
 (b) A set of vectors that are linearly dependent. After adding one more vector, the set becomes linearly independent.  
 (c) A set of vectors in  $\mathbb{R}^3$  with the following properties (four possibilities):

spans $\mathbb{R}^3$ , linearly independent	spans $\mathbb{R}^3$ , linearly dependent
doesn't span $\mathbb{R}^3$ , linearly independent	doesn't span $\mathbb{R}^3$ , linearly dependent

For each case that is *possible*, how many vectors could be in the set? (State any constraints, as in “there must be at least...” or “at most...”)

- (e) A system of equations with a unique solution. After adding another equation to the system, the new system has infinitely-many solutions.  
 (f) \* A system of equations without any solutions. After deleting an equation, the system has infinitely-many solutions.

**Solution by Groups A10, B10, C10 – due in class on Wednesday 1/17**