Math 308 E Test Prep Problems - Week #1

(1) Before paying employee bonuses and state and federal taxes, a company earns profits of $103,000. The company pays employees a bonus equal to 5% of after-tax profits. State tax is 5% of profits (after bonuses are paid). Finally, federal tax is 40% of profits (after bonuses and state tax are paid). Calculate the amounts paid in bonuses, state tax and federal tax.

Solution by Groups A1, B1, C1 - due in class on Friday 1/5

(2) (Geometry Question) For each part below, give an example a linear system of equations in two variables that has the given property. In each case, draw the lines corresponding to the equations in the system.

(a) has no solution
(b) has exactly one solution
(c) has infinitely many solutions

(i) Add or remove equations in (b) to make an inconsistent system.
(ii) Add or remove equations in (b) to create infinitely many solutions.
(iii) Add or remove equations in (b) so that the solution space remains unchanged.
(iv) Can you add or remove equations in (b) to change the unique solution you had to a different unique solution?

In each of (i) - (iv) justify your action in words.

Solution by Groups A2, B2, C2 - due in class on Monday 1/8

(3) (a) Use Gauss-Jordan elimination to find the general solution for the following system of linear equations:

$$\begin{aligned} z_2 + 3z_3 - z_4 &= 0 \\ -z_1 - z_2 - z_3 + z_4 &= 0 \\ -2z_1 - 4z_2 + 4z_3 - 2z_4 &= 0 \end{aligned}$$

(b) Give an example of a solution to the previous system of linear equations.
(c) The points $(1, 0, 3)$, $(1, 1, 1)$, and $(-2, -1, 2)$ lie on a unique plane $a_1x_1 + a_2x_2 + a_3x_3 = b$. Using your previous answers, find an equation for this plane. (Hint: think about the relationship between the previous system and the one you would need to solve in this question.)

Solution by Groups A3, B3, C3 - due in class on Monday 1/8

(4) (Interpolating polynomials) Say we want to find a polynomial $f(x)$ of degree 3,

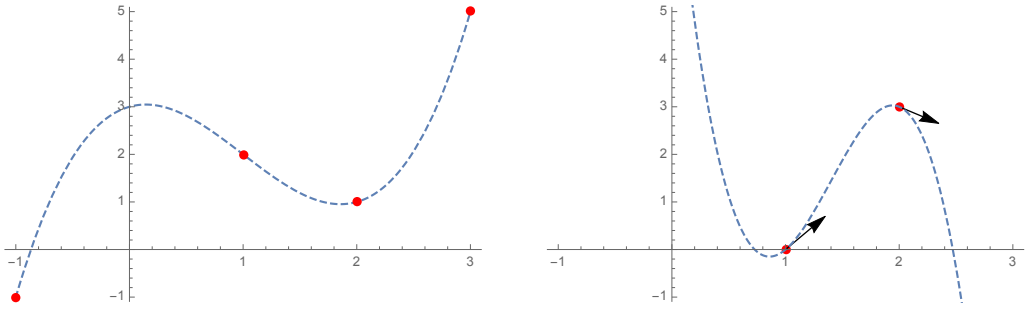
$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3,$$

satisfying some interpolation conditions. In each case below, write a system of linear equations whose solutions are (a_0, a_1, a_2, a_3). You don't need to solve.

(a) We want $f(x)$ to pass through the points $(-1, -1), (1, 2), (2, 1)$ and $(3, 5)$.

(b) We want $f(x)$ to pass through $(1, 0)$ with derivative $+2$ and $(2, 3)$ with derivative -1.

Graphically:



(c) (Discuss) What if we had more than four points to consider? Fewer?

(d) (Discuss) Can we still use linear algebra if $f(x)$ is another kind of function, such as $f(x) = a\sin(x) + b\cos(x)$? $f(x) = ae^{bx}$?

Solution by Groups A4, B4, C4 - due in class on Monday 1/8

All students should turn in solutions to the above four problems in TA section on Tuesday 1/9