

## 2.2-2 2.2-3 Summary

#rows    #cols

Given

$$u_1, \dots, u_m \in \mathbb{R}^n$$

$$A = [u_1 \dots u_m] \quad \begin{matrix} \downarrow \\ n \times m \end{matrix} \quad \text{matrix}$$

B - echelon form of A

$$\underbrace{u_1 x_1 + \dots + u_m x_m}_A = b$$

$$Ax = b \quad \leftrightarrow \quad \text{system of linear equations.}$$

(I)

$\text{Span}\{u_1, \dots, u_m\} = \mathbb{R}^n \Leftrightarrow$  every  $b \in \mathbb{R}^n$  is a linear combination of  $u_1, \dots, u_m$

$\Leftrightarrow Ax = b$  has a solution for every  $b$

$\Leftrightarrow$  every row of B has a pivot position

(II)

$u_1, \dots, u_m$  LI  $\Leftrightarrow$  the only sol<sup>n</sup> to  $Ax = 0$  is

$$x_1 = 0, x_2 = 0, \dots, x_m = 0$$

$\Leftrightarrow$  no  $u_i$  is in the span of the rest

$\Leftrightarrow$  every column of B has a pivot position

$\Leftrightarrow Ax = b$  has a unique sol<sup>n</sup> whenever it has a sol<sup>n</sup>

(III)

$u_1, \dots, u_m$  LD  $\Leftrightarrow$  there are non-zero sol<sup>n</sup>s to

$$Ax = 0$$

$\Leftrightarrow$  some  $u_i$  is in the span of the rest

$\Leftrightarrow$  some col of B is missing a pivot position

$\Leftrightarrow Ax = b$  has infinitely many sol<sup>n</sup>s whenever it has a sol<sup>n</sup>

Other facts

① some  $u_i = 0 \Rightarrow \{u_1, \dots, u_m\}$  LD

②  $m > n \Rightarrow u_1, \dots, u_m$  LD

- Know why these facts are true
- Know how to apply them to problems.