1. Suppose \( f : D \to \mathbb{R} \) and \( x_0 \) is a limit point of \( D \). Prove that \( \lim_{x \to x_0} f(x) = \ell \) if and only if, for every \( \epsilon > 0 \), there exists a \( \delta > 0 \) such that, if \( x \in D \) and \( 0 < |x - x_0| < \delta \), then \( |f(x) - \ell| < \epsilon \).

2. Let \( n \) be a natural number. Determine \( \lim_{x \to 1} \frac{x^n - 1}{x - 1} \) and prove you are correct.

   (You may find the Difference of Powers Formula useful: If \( n \in \mathbb{N} \) and \( a,b \in \mathbb{R} \), then \( a^n - b^n = (a - b) \sum_{k=0}^{n-1} a^{n-1-k}b^k \).)

3. Prove the Squeeze Theorem for functions: Suppose \( f : D \to \mathbb{R} \), \( g : D \to \mathbb{R} \), \( h : D \to \mathbb{R} \), \( x_0 \) is a limit point of \( D \), and \( f(x) \leq g(x) \leq h(x) \) for all \( x \in D \setminus \{x_0\} \). Prove that, if \( \lim_{x \to x_0} f(x) = \lim_{x \to x_0} h(x) = \ell \), then \( \lim_{x \to x_0} g(x) = \ell \). (You may use the Squeeze Theorem for sequences.)

4. Let \( f : \mathbb{R} \to \mathbb{R} \).

   (a) Give an \( \epsilon-\delta \) proof that, if \( \lim_{x \to x_0} f(x) = \ell \), then \( \lim_{x \to x_0} |f(x)| = |\ell| \).

   (b) Give an example of a function \( f(x) \) and an \( \ell \in \mathbb{R} \) such that \( \lim_{x \to x_0} |f(x)| = |\ell| \) but \( \lim_{x \to x_0} f(x) \neq \ell \).

   (c) Give an \( \epsilon-\delta \) proof that \( \lim_{x \to x_0} f(x) = 0 \) if and only if \( \lim_{x \to x_0} |f(x)| = 0 \).

5. Suppose \( f : \mathbb{R} \to \mathbb{R} \) and there exists a real constant \( M > 0 \) such that \( |f(x)| \leq Mx^2 \) for all \( x \). Prove that \( \lim_{x \to 0} \frac{f(x)}{x} = 0 \).

6. Suppose \( m_1 \) and \( m_2 \) are real constants such that \( m_1 \neq m_2 \). Define \( f : \mathbb{R} \to \mathbb{R} \) by

   \[
   f(x) = \begin{cases} 
   m_1x + 3 & \text{if } x < 0 \\
   m_2x + 3 & \text{if } x \geq 0.
   \end{cases}
   \]

   Prove that \( f(x) \) is continuous but not differentiable at 0.

7. Define \( f : \mathbb{R} \to \mathbb{R} \) by

   \[
   f(x) = \begin{cases} 
   x^2 & \text{if } x < 0 \\
   x & \text{if } x \geq 0.
   \end{cases}
   \]

   Is \( f(x) \) differentiable at 0? Prove you are correct.

8. Suppose \( f : \mathbb{R} \to \mathbb{R} \) and \(-x^2 \leq f(x) \leq x^2 \) for all \( x \). Prove that \( f \) is differentiable at 0 and that \( f'(0) = 0 \).