1. Define $f : \mathbb{R} \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} 
-x & \text{if } x < 1 \\
1 - x & \text{if } 1 \leq x \leq 4 \\
\frac{-19}{16}x^2 + 4x & \text{if } x > 4.
\end{cases}$$

Determine the values at which $f$ is continuous. Prove you are correct.

2. Give an example of each of the following. Give an explicit formula for each function and sketch its graph to demonstrate that your answer is correct.

   (a) a function $f : [0, 1] \rightarrow \mathbb{R}$ that has no maximum
   (b) a continuous function $f : (0, 1) \rightarrow \mathbb{R}$ that is unbounded
   (c) a continuous function $f : (0, 1) \rightarrow \mathbb{R}$ that is bounded but has no minimum
   (d) a continuous function $f : [0, \infty) \rightarrow \mathbb{R}$ that is unbounded
   (e) a continuous function $f : [0, \infty) \rightarrow \mathbb{R}$ that is bounded but has no minimum
   (f) a function $f : [0, 1] \rightarrow \mathbb{R}$ that does not take on every value between $f(0)$ and $f(1)$

3. Suppose $f$ and $g$ are continuous on the interval $[a, b]$ and that $f(a) < g(a)$ and $f(b) > g(b)$. Prove that there is a real number $c \in (a, b)$ such that $f(c) = g(c)$.

4. Suppose $f : [0, 1] \rightarrow [0, 1]$ is continuous. Prove that $f$ has a fixed point: a real number $c \in [0, 1]$ such that $f(c) = c$. (HINT: Consider the function $h(x) = f(x) - x$.)