1. Determine whether the statement is TRUE or FALSE. Justify your answer with a proof or counterexample.
   (a) If \( \{a_n\} \) converges to \( a \) and \( c < a_n < d \) for all \( n \in \mathbb{N} \), then \( c < a < d \).
   (b) The sequence \( \{n - 5\} \) converges.
   (c) If \( \{a_n\} \) and \( \{b_n\} \) are monotone, then \( \{a_n + b_n\} \) is monotone.
   (d) If \( \{a_n\} \) and \( \{b_n\} \) are monotone, then \( \{a_nb_n\} \) is monotone.
   (e) If \( \{a_n\} \) is bounded, then \( \{a_n\} \) converges.
   (f) If \( \{a_n\} \) is monotone, then \( \{a_n\} \) converges.
   (g) Every subsequence of a bounded sequence is bounded.
   (h) Every subsequence of a monotone sequence is monotone.
   (i) Every sequence of non-negative real numbers has a convergent subsequence.
   (j) If \( \{a_n\} \) has a convergent subsequence, then \( \{a_n\} \) converges.

2. Determine whether the sequence is monotonic. Prove you are correct.
   (a) \( \{n^2\} \)
   (b) \( \left\{ \frac{1}{\sqrt{n + 2}} \right\} \)
   (c) \( \left\{ 1 + \left( -\frac{1}{n} \right)^n \right\} \)
   (d) \( \left\{ \frac{n}{2^n} \right\} \)
   (e) \( \left\{ \frac{n}{n + 1} \right\} \)
   (f) \( \left\{ \frac{1}{n^2} + \left( -\frac{1}{3} \right)^n \right\} \)

3. Suppose \( \{a_n\} \) is monotonic. Prove that \( \{a_n\} \) converges if and only if \( \{a_n^2\} \) converges.

4. Suppose that \( \{a_n\} \) converges to \( a \) and \( |a| < 1 \). Prove that \( \{(a_n)^n\} \) converges to 0.
   (NOTE: You may not simply apply the Power Property for Limits. Why not? Also, the theorem proved in class that, if \( |c| < 1 \), then \( \lim c^n = 0 \), does not apply directly (though it may prove useful). Why not?)

5. Suppose \( x \) is a real number such that \( 0 < |x| < 1 \).
   (a) Show that there is a \( y > 0 \) such that \( |x| = \frac{1}{1 + y} \).
   (b) Prove that \( |x^n| \leq \frac{1}{ny} \) for all \( n \in \mathbb{N} \).
   (c) Prove that \( \{\sqrt[n]{x^n}\} \) converges to 0.
   (d) Prove that, if \( 0 < u < 1 \), then \( \{nu^n\} \) converges to 0.
      [HINT: For all \( n \), \( nu^n = (\sqrt[n]{u^n/2})^2 \).]