1. (a) \( \frac{dy}{dx} = \frac{1}{x^3e^{-x}}[x^3(-e^{-x}) + e^{-x}(3x^2)] \)

(b) \( \frac{dy}{dx} = x^2 \cdot \frac{1}{1 + 5e^x} \cdot 5e^x + 2x \ln(1 + 5e^x) \)

(c) \( \frac{dy}{dx} = \frac{(1 + x)e^x - e^x}{(1 + x)^2} \)

(d) \( \frac{dy}{dx} = \frac{(x^2 + 5x) \cdot \frac{1}{2x + 7} \cdot 2 - [\ln(2x + 7)] \cdot (2x + 5)}{(x^2 + 5x)^2} \)

(e) \( \frac{dy}{dx} = \frac{1}{2}(2x + 6)^{-1/2} \cdot 2 = \frac{1}{\sqrt{2x + 6}} \)

(f) \( \frac{dy}{dt} = (3t^2 + 7)^4 \cdot 7(t^3 - t)^6(3t^2 - 1) + (t^3 - t)^7 \cdot 4(3t^2 + 7)^3(6t) \)

(g) \( \frac{dy}{dx} = 7 \left[ x - (x + 3)^4 \right]^6 [1 - 4(x + 3)^3] \)

(h) \( \frac{dy}{dx} = e^{5x} \cdot 2(x^3 + 4)(3x^2) + (x^3 + 4)^2 \cdot e^{5x} \cdot 5 \)

2. (a) The maximum value is \(-1\).

(b) The maximum value is \(\frac{1}{3}\).

(c) \( g(x) \) has no critical points since \( g'(x) = f''(x) \) is never 0.

3. The PRINTED MATTER will have maximal area when \( x = \sqrt{500} \) and \( y = 2\sqrt{500} \).

4. (a) \( f'(x) = -2xe^{-x^2} \) and \( f''(x) = 2e^{-x^2}(2x^2 - 1) \) (simplified)

(b) \( f'(x) = 0 \) only at \( x = 0 \).

(c) \( f(x) \) is increasing on \(( -\infty, 0)\) and decreasing on \((0, \infty)\). \( f(x) \) is concave up on \( (-\infty, -\sqrt{\frac{1}{2}}) \) and on \( \left( \sqrt{\frac{1}{2}}, \infty \right) \), concave down on \( \left( -\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}} \right) \).

(d) The \( y \)-intercept is \( f(0) = 1 \). Since \( f(x) \) is never 0, there is no \( x \)-intercept.

(e) The limits are both equal to 0. This means that the graph of \( f(x) \) will approach the line \( y = 0 \) (the \( x \)-axis) as \( x \) approaches \( \pm \infty \). That is, the \( x \)-axis is a horizontal asymptote for the graph of \( f(x) \).

5. (a) \(-1\)

(b) \(-\frac{1}{6}\)

(c) \(-5\)

(d) 5

(e) 1