1. Suppose you draw two cards from a standard deck, one-at-a-time, without replacement.

   (a) What is the probability that the second card is red given that the first card is red?

   ANSWER: \( P(\text{red on second}|\text{red on first}) = \frac{25}{51} \)

   (b) What is the probability that the second card is a 10, given that the first card is a diamond?

   ANSWER: Let \( \Diamond_1 \) represent the event that we draw a diamond on the first card and let \( 10_2 \) represent the event that we draw a 10 on the second. Then,

   \[
P(10_2|\Diamond_1) = \frac{P(10_2 \cap \Diamond_1)}{P(\Diamond_1)}.
   \]

   But the intersection of \( 10_2 \) and \( \Diamond_1 \) is the union of two disjoint events: the event that we draw the ten of diamonds on the first draw, followed by another 10, and the event that we draw any other diamond on the first draw, followed by a 10. The probability of drawing the ten of diamonds on the first draw, followed by another 10, is \( \frac{1}{52} \cdot \frac{3}{51} \). The probability of drawing any other diamond, followed by a ten, is \( \frac{12}{52} \cdot \frac{4}{51} \). So,

   \[P(10_2 \cap \Diamond_1) = \frac{1}{52} \cdot \frac{3}{51} + \frac{12}{52} \cdot \frac{4}{51} = \frac{1}{52}.
   \]

   The probability of drawing a diamond on the first card is \( P(\Diamond_1) = \frac{13}{52} \) and thus

   \[P(10_2|\Diamond_1) = \frac{1/52}{13/52} = \frac{1}{13}.
   \]

2. A deck of ten cards consists of the following:

   \[A\spadesuit, 2\spadesuit, 3\spadesuit, 4\spadesuit, 5\spadesuit, A\heartsuit, 2\heartsuit, 3\heartsuit, A\clubsuit, 2\clubsuit.\]

   You draw one card from this deck of ten. You draw a card and see that it’s not an ace (\(A\)). What is the probability that it is a spade (\(\spadesuit\)), given that it’s not an ace?

   ANSWER: \(\frac{4}{7}\)

3. A shipment contains 10,000 clams. Of these, 1000 are bad.
(a) An inspector selects one clam at random and tests it for badness. What is the probability that the selected clam is not bad?

ANSWER: \( \frac{9000}{10000} = 0.9 \)

(b) The inspector draws one clam and it is bad. Without replacing the bad clam, the inspector draws another. What is the probability that this clam is also bad?

ANSWER: \( \frac{999}{9999} = 0.0999 \)

4. Subjects in a medical study were classified as having one of five different types of cancer (breast, endometrial, cervix, lung, and melanoma). Each disease was further classified as in situ (confined to the point of origin) or invasive (spread to other tissues). The following table shows the number of subjects in each classification.

<table>
<thead>
<tr>
<th>Type of Cancer</th>
<th>In Situ</th>
<th>Invasive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breast</td>
<td>507</td>
<td>2596</td>
</tr>
<tr>
<td>Endometrial</td>
<td>783</td>
<td>54</td>
</tr>
<tr>
<td>Cervix</td>
<td>433</td>
<td>98</td>
</tr>
<tr>
<td>Lung</td>
<td>0</td>
<td>135</td>
</tr>
<tr>
<td>Melanoma</td>
<td>165</td>
<td>409</td>
</tr>
</tbody>
</table>

(a) A subject in the study is chosen at random. Find each of the following.

i. the probability that the subject has invasive cancer, given that she has cancer of the cervix

**ANSWER:** \( P(\text{invasive}| \text{cervix}) = \frac{98}{531} \)

ii. the probability that the subject has lung cancer, given that the cancer is in situ

**ANSWER:** \( P(\text{lung}| \text{in situ}) = 0 \)

iii. the probability that the subject has breast cancer or in situ cancer

**ANSWER:** \( P(\text{breast} \cup \text{in situ}) = P(\text{breast}) + P(\text{in situ}) - P(\text{breast} \cap \text{in situ}) = \frac{4484}{5180} \)

(b) Are the events “the subject has breast cancer” and “the subject has in situ cancer” independent events? Explain.

**ANSWER:** No! Let \( B \) be the event that the chosen subject has breast cancer and let \( I \) be the event that the chosen subject has in situ cancer. Then \( P(B \cap I) = \frac{507}{5180} = 0.0970 \). But \( P(B) = \frac{3103}{5180} \) and \( P(I) = \frac{1888}{5180} \), which means that \( P(B \times P(I) = \frac{3103 \times 1888}{5180^2} = 0.2183 \). Since \( P(B \cap I) \neq P(B) \times P(I) \), the events \( B \) and \( I \) are not independent.

5. Huntington’s Chorea is a serious degenerative disease of the nervous system that is caused by the presence of a dominant allele \( D \). People who have the \( D \) allele
usually begin to show symptoms between the ages of 40 and 50. Joseph has just been diagnosed with Huntington’s. Since his mother did not have the disease, this means that Joseph’s genotype is $Dd$. The genotype of Joseph’s wife Delilah is unknown, but the probability that Delilah has the $D$ allele is 0.00008. What is the probability that Delilah carries the $D$ allele if she and Joseph have five children who are genotype $dd$?

**SOLUTION:** Use Bayes’ Theorem:

$$P(\text{Delilah is } Dd | \text{five } dd \text{ children}) = \frac{P(\text{five } dd \text{ children} | \text{Delilah is } Dd) \times P(\text{Delilah is } Dd)}{P(\text{five } dd \text{ children})}.$$ 

To get the denominator, you must use the partition theorem:

$$P(\text{five } dd \text{ children}) = P(\text{Delilah is } Dd) \times P(\text{five } dd \text{ children} | \text{Delilah is } Dd) + P(\text{Delilah is } dd) \times P(\text{five } dd \text{ children} | \text{Delilah is } dd) = 0.00008 \times \left(\frac{1}{4}\right)^5 + (1 - 0.00008) \times \left(\frac{1}{2}\right)^5 = 0.031247578.$$ 

So, $P(\text{Delilah is } Dd | \text{five } dd \text{ children}) = \frac{0.000976563 \times 0.00008}{0.031247578}.$