This is by no means an exhaustive review of the topics we’ve covered. Any topic covered in class or on the homework is fair game for the midterm exam. This should give you an idea of the length and difficulty of the exam.

1. Find the limit, if it exists, or show that the limit does not exist:
   \[ \lim_{(x,y)\to(0,0)} \frac{6x^3y}{2x^4 + y^4}. \]

2. Let \( f : \mathbb{R} \to \mathbb{R} \) be a continuously differentiable function and consider the surface \( z = xf\left(\frac{x}{y}\right) \). Show that the origin lies on every tangent plane to this surface.

3. A metal plate is situated on the \( xy \)-plane in such a way so that the temperature \( T \) at a point is inversely proportional to its distance from the origin. The temperature at the point \( P(3,4) \) is 100º.
   
   (a) Find the rate of change of \( T \) at the point \( P \) in the direction of the vector \( v = i + j \).
   (b) In what direction does the temperature increase most rapidly at \( P \)?
   (c) Give a unit vector \( u \) such that the rate of change of temperature at \( P \) in the direction of \( u \) is 0.

4. Find the point on the sphere \( x^2 + y^2 + z^2 = 9 \) that is closest to the point \( (2,3,4) \).

5. Let \( n \) be a positive integer. The function \( f : \mathbb{R} \to \mathbb{R} \) is called homogeneous of degree \( n \) if it satisfies the equation
   \[ f(tx,ty) = t^n f(x,y) \]
   for all real \( t \).
   
   (a) Verify that \( f(x,y) = x^2y + 2xy^2 + 5y^3 \) is homogeneous of degree 3.
   (b) Show that if \( f \) is a differentiable homogeneous function of degree \( n \), then
   \[ f_x(tx,ty) = t^{n-1}f_x(x,y) \]
   for all real \( t \).
   (c) Show that if \( f \) is a continuously differentiable homogeneous function of degree \( n \), then
   \[ x \cdot f_x(x,y) + y \cdot f_y(x,y) = n \cdot f(x,y). \]