• Determinants
  – Be able to compute determinants (minor matrices, cofactor expansions, etc.).
  – Know the properties of determinants:
    * \( \det(AB) = \det(A)\det(B) \)
    * \( \det(A) = 0 \) if and only if \( A \) is singular
    * \( \det(A^{-1}) = \frac{1}{\det(A)} \) if \( A \) is non-singular
    * \( \det(A^T) = \det(A) \)
    * \( \det(T) = t_{11}t_{22}...t_{nn} \) if \( T \) is an \( n \times n \) triangular matrix
  – Know the effects of the elementary row operations on determinants.

• The Eigenvalue Problem
  – Be able to find (real and complex) eigenvalues and eigenvectors of a given matrix.
  – Know all about the characteristic polynomial of a matrix.
  – Understand algebraic and geometric multiplicities and defective matrices.
  – Know how to diagonalize a matrix.
  – Know how to use eigenvalues and eigenvectors to solve initial value problems involving a system of differential equations.

• General Vector Spaces
  – Know the big examples: \( \mathbb{R}^n \), \( M_{m \times n} \), \( \mathcal{P}_n \), \( \mathcal{C}[a,b] \)
  – Know the properties of vector spaces listed on page 365.
  – Understand subspaces, spanning sets, linear independence, bases, and dimension.
  – Know how to find the coordinate vectors with respect to a given basis.

• The Big Results
  – Let \( u_1, ..., u_k \) be eigenvectors of an \( n \times n \) matrix \( A \) corresponding to distinct eigenvalues \( \lambda_1, ..., \lambda_k \). Then \( \{u_1, ..., u_k\} \) is linearly independent.
  – Let \( A \) be a real \( n \times n \) matrix with eigenvalue \( \lambda \) and corresponding eigenvector \( x \). Then \( \lambda \) is also an eigenvalue of \( A \) with corresponding eigenvector \( x \).
  – Similar matrices have the same eigenvalues with the same algebraic multiplicity.
  – An \( n \times n \) matrix \( A \) is diagonalizable if and only if \( A \) possesses a linearly independent set of \( n \) eigenvectors.
  – Let \( V \) be a vector space with a basis \( B = \{v_1, ..., v_p\} \). Let \( S = \{u_1, ..., u_m\} \) be a subset of \( V \) and let \( T = \{[u_1]_B, ..., [u_m]_B\} \). Then \( S \) is a basis for \( V \) if and only if \( T \) is a basis for \( \mathbb{R}^p \).

  Let \( V \) be a vector space with dimension \( p \).
  * Any set of \( p + 1 \) or more vectors in \( V \) is linearly dependent.
  * Any set of fewer than \( p \) vectors in \( V \) does not span \( W \).
  * Any set of \( p \) linearly independent vectors in \( V \) is a basis for \( V \).
  * Any set of \( p \) vectors that spans \( V \) is a basis for \( V \).