Make sure you complete Activity 8 by Monday.
Income Flow: If total income from a continuous income stream has an annual rate of flow given by a function \( r(t) \), total income after \( k \) years is given by

\[
I(k) = \int_0^k r(t) \, dt.
\]

Examples:

1. Annual rate of flow is \( 8000 e^{0.05t} \) dollars per year.
   
   Total income in first 5 years is
   
   \[
   \int_0^5 8000 e^{0.05t} \, dt = \left. \frac{8000 e^{0.05t}}{0.05} \right|_0^5
   \]
   
   \[
   = 160,000 (e^{0.3} - 1) \approx \$55977.41
   \]

2. Careful about units.
   
   Suppose rate is \( 100 e^{0.02t} \) dollars per month.
   
   Total income in first 5 years.
   
   Total income = \[
   \int_0^{5 \times 12} 100 e^{0.02t} \, dt
   \]
   
   \[
   = \ldots \approx \$11600.58
   \]
Consumer Surplus

Reminder: A product's demand function relates price per unit $p$ to the number of units $x$ that consumers are willing to purchase at that price. The supply function relates price per unit $p$ to the number of units $x$ that manufacturers are willing to sell at that price.

Market equilibrium occurs when the supply and demand curves intersect.
Suppose demand for a product is given by the function
\[ P = 5000 - x^2. \]

If equilibrium price is \$1400/unit, what is equilibrium quantity?

\[ 1400 = 5000 - x^2 \]
\[ x^2 = 3600 \]
\[ x = 60 \]

If this product sells for \$1400 per unit, consumers will purchase 60 units.

But there are some consumers who are willing to spend more than \$1400 on this product. Take Jody - Jody is willing to pay up to \$2000 for this product. Jody has a personal surplus of \$2000 - 1400 = \$600 with respect to this product.
If we add up personal surplus for all consumers, we the consumer surplus for this product, which is equal to the shaded area below:

\[ CS = \left( \text{area under the demand curve from 0 to } 60 \right) - \left( \text{area of rectangle} \right) \]

Area under demand curve from 0 to 60:

\[
\int_0^{60} (5000 - x^2) \, dx = \left[ 5000x - \frac{1}{3}x^3 \right]_0^{60} = 5000 \cdot 60 - \frac{60^3}{3} = 228,000
\]

Area of rectangle:

\[ 1400 \dollar / \text{unit} \cdot 60 \text{ units} = 84,000 \]

\[ CS = 228,000 - 84,000 = \$144,000 \]
In general, if demand is given by a function \( p = f(x) \) and the equilibrium quantity is \( x = q \), then

\[
\text{area} = CS = \left( \int_0^q f(x) \, dx \right) - \left( q \cdot f(q) \right)
\]

4) Suppose supply is given by \( p = 3 + 0.1x \) and demand is given by \( p = \frac{48}{x+2} \). Find consumer surplus under pure competition. (i.e., at equilibrium)
To find equilibrium point, set
\[ 3 + 0.1x = \frac{48}{x+2} \]
and solve for \( x \).

\[(x+2)(3+0.1x) = 48 \]
\[ 3x + 6 + 0.1x^2 + 0.2x = 48 \]
\[ 0.1x^2 + 3.2x + 6 = 48 \]
\[ 0.1x^2 + 3.2x - 42 = 0 \]

\[ x = 10 \text{ and something negative} \]

\[ \text{Suppose } p = 0.1x + 3 \]

The graph shows the intersection of supply and demand functions.

The area under demand is given by the integral:
\[ \int_0^{10} \frac{48}{x+2} \, dx = 48 \ln(x+2) \bigg|_0^{10} \]
\[ = 48(\ln 12 - \ln 2) \]
\[ = 86.00445452 \]

The area of the rectangle is:
\[ TR(10) = 10 \cdot 4 = 40 \]

The consumer surplus is:
\[ CS = 86.00445452 - 40 = \boxed{\$46.00} \]

\[ \text{NEXT TIME: Producer's surplus + brief into multi-variable functions.} \]