Area under $MC(q)$

Example: $MC(q) = 10\sqrt{q + 9}$

a) Find $\int MC(q) \, dq$.

$\int 10(q+9)^{1/2} \, dq = 10 \cdot \frac{2}{3} (q+9)^{3/2} + K$

$= \frac{20}{3} (q+9)^{3/2} + K$

b) Find $VC(q)$.

We know $VC(0) = 0$ and $VC(q) = \frac{20}{3} (q+9)^{3/2} + K$ for some $K$.

$VC(0) = \frac{20}{3} (0+9)^{3/2} + K = 180 + K$

So, $VC(0) = 0$ and $VC(0) = 180 + K$.

$180 + K = 0$ and $K = -180$

$VC(q) = \frac{20}{3} (q+9)^{3/2} - 180$
c) Find the area under MC from $q = 0$ to $q = 27$.

$$\text{area} = \int_{0}^{27} \frac{20}{3} (27+q)^{3/2} \, dq = \frac{20}{3} \left[ 1260 \right] = 1260$$

OR

$$\int_{0}^{27} MC(q) \, dq = \int_{0}^{27} 10(q+9)^{1/2} \, dq = \left[ \frac{20}{3} (q+9)^{3/2} \right]_{0}^{27} = \left[ \frac{20}{3} (36)^{3/2} \right] - \left[ \frac{20}{3} (9)^{3/2} \right] = 1260$$

d) Compute the change in TC if $q$ increases from $27$ to $72$.

Want: $TC(72) - TC(27)$

$$TC(q) = VC(q) + FC$$

$$TC(72) - TC(27) = (VC(72) + FC) - (VC(27) + FC)$$

$$= VC(72) - VC(27) = \left( \text{area under MC from } q = 27 \text{ to } q = 72 \right) = \int_{27}^{72} MC(q) \, dq = \left[ \frac{20}{3} (q+9)^{3/2} \right] = 3420$$

Punchline:

$$\int_{a}^{b} MC(q) \, dq = (\text{area under MC from } q = a \text{ to } q = b) = TC(b) - TC(a) = VC(b) - VC(a)$$
Review for Exam II

- Given the graph or formula for a function \( f(x) \), be able to identify:
  - critical values (a.k.a. critical numbers)
    of \( f(x) \) (x-values)
  - critical points \( (x, y) \) of \( f(x) \)
  - relative (local) optima of \( f(x) \)
  - absolute (global) optima of \( f(x) \)
    on an interval
  - intervals on which \( f(x) \) is increasing
    or decreasing
  - intervals on which \( f(x) \) is concave up
    or concave down
  - points of inflection and horizontal
    points of inflection ← "seats"

- Given the graph or formula for a function \( f'(x) \), be able to identify:
  - critical values of \( f(k) \)
  - relative optima of \( f(k) \)
  - p.o.i. and h.p.o.i. of \( f(k) \)
  - intervals on which \( f(k) \) is increasing,
    decreasing, concave up, concave down
Given the formula for $f(x)$ or $f'(x)$, be able to apply the Second Derivative Test to determine whether $f(x)$ has a local max or a local min at a specific value of $x$.

Be able to take derivatives of crazy functions.

Be able to compute indefinite integrals — don't forget the $+C$.

Given the graph of $f(x)$, be able to compute $\int_a^b f(x)\,dx$ using areas.

Given the formula for $f(x)$, be able to compute $\int_a^b f(x)\,dx$ using the Fundamental Theorem.
- Be able to apply and apply these concepts in context.