Office Hours today only:
1:00 - 3:30 pm

Exam II info on-line
Exam is Tues in Quiz Section
covers everything since
the last midterm:
3/10/22 - 13.2
Computing Definite Integrals (§13.2)

Recall: An indefinite integral \( \int f(x) \, dx \) is a function:
\[ \int f(x) \, dx = \text{the general anti-derivative of } f(x) \]

A definite integral \( \int_a^b f(x) \, dx \) is a number:
\[ \int_a^b f(x) \, dx = \text{area of } \int_{a}^{b} f(x) \, dx = \text{area under } f(x) \text{ from } a \text{ to } b \]

The idea that area can be connected to a derivative is powerful and profound and is known as the Fundamental Theorem of Calculus, which says:

To compute \( \int_a^b f(x) \, dx \):

Step 1: Find the most obvious anti-derivative of \( f(x) \). Call it \( F(x) \).

Step 2: Evaluate \( F(x) \) at \( b \) and again at \( a \).

Step 3: Subtract:
\[ \int_a^b f(x) \, dx = F(x) \bigg|_a^b = F(b) - F(a) \]

Read \( F(x) \) evaluated from \( a \) to \( b \).
Examples: ① \[ \int_0^2 (3x^2 - 4x + 5) \, dx \]

Step 1: \[ F(x) = x^3 - 2x^2 + 5x \]

Step 2: \[ F(2) = 2^3 - 2 \cdot 2^2 + 5 \cdot 2 = 10 \]
\[ F(0) = 0^3 - 2 \cdot 0^2 + 5 \cdot 0 = 0 \]

Step 3: \[ \int_0^2 (3x^2 - 4x + 5) \, dx = \left[ x^3 - 2x^2 + 5x \right]_0^2 = 10 - 0 = 10 \]
2) \( \int_1^5 \frac{3}{4x^2} \, dx = \int_1^5 \frac{3}{4} x^{-2} \, dx \)

**Step 1:** 
\[ F(x) = \frac{3}{4} (-x^{-1}) = \frac{-3}{4x} \]

**Step 2:** 
\[ F(5) = \frac{-3}{20} \]
\[ F(1) = \frac{-3}{4} \]

**Step 3:** 
\[
\int_1^5 \frac{3}{4x^2} \, dx = \frac{-3}{4x} \bigg|_1^5 = \frac{-3}{20} - \left( \frac{-3}{4} \right) \\
= \frac{-3}{20} + \frac{3 \cdot 5}{4} \\
= \frac{-3 + 15}{20} = \frac{12}{20} = \frac{3}{5}
\]

\[ y = \frac{3}{4x^2} \]

Area = \( \frac{3}{5} \)
\[ \int_{-1}^{0} x^4 + x \, dx = \left[ \frac{1}{5}x^5 + \frac{1}{2}x^2 \right]_{-1}^{0} \]

\[ = \left( \frac{1}{5} \cdot 0^5 + \frac{1}{2} \cdot 0^2 \right) - \left( \frac{1}{5} (-1)^5 + \frac{1}{2} (-1)^2 \right) \]

\[ = 0 + \frac{1}{5} - \frac{1}{2} = \frac{1}{5} - \frac{1}{2} = \frac{2}{10} - \frac{5}{10} = \frac{-3}{10} \]

- (area of \( y = x^4 + x \)) = \frac{-3}{10}
4. \( \int_{1}^{3} 8x - 1 \, dx = \left[ 4x^2 - x \right]_{1}^{3} = (4 \cdot 9 - 3) - (4 \cdot 1 - 1) = 30 \)

5. \( \int_{0}^{1} e^{x/10} \, dx = \left. 10e^{x/10} \right|_{0}^{1} = 10e^{1/10} - 10e^{0} = 10e^{1/10} - 10 \approx 1.05 \)

6. Reminder: \( \ln e^n = n \) and \( e^{\ln n} = n \)

\[ \int_{e^4}^{1} \frac{1}{u} \, du = \ln u \bigg|_{e^4}^{1} = \ln e^4 - \ln e = 4 - 1 = 3 \]
7) Suppose \( MR(q) = 15 - \frac{1}{2} q \).

a) Compute \( \int_0^4 MR(q) \, dq \) and interpret the answer.

\[
\int_0^4 MR(q) \, dq = \int_0^4 15 - \frac{1}{2} q \, dq = \left[ 15q - \frac{1}{4} q^2 \right]_0^4
\]

\[
= (15 \cdot 4 - \frac{1}{4} \cdot 16) - (15 \cdot 0 - \frac{1}{4} \cdot 0^2)
\]

\[
= 60 - 4 = 56
\]

The area = 56 = TR(4)

Total revenue for selling \( q = 4 \) is 56.
(b) Compute \( \int_{2}^{8} MR(q) \, dq \) and interpret the result.

\[
\int_{2}^{8} MR(q) \, dq = \int_{2}^{8} 15 - \frac{1}{2}q \, dq = \left[ 15q - \frac{1}{4}q^2 \right]_{2}^{8}
\]

\[
= \left( 15 \cdot 8 - \frac{1}{4} \cdot 64 \right) - \left( 15 \cdot 2 - \frac{1}{4} \cdot 4 \right) = (120 - 16) - (30 - 1) = 75
\]

MR(q) = 15 - \frac{1}{2}q

Area = \( \int_{2}^{8} MR(q) \, dq \) = \( \text{area under } MR \) from 0 to 8 - \( \text{area under } MR \) from 0 to 2

This is the change in TR if quantity changes from 2 to 8.