Lecture 1 - Modelling

Note Title 1/3/2020

Class Web Page

https://sites.math.washington.edu/~sylvest/courses/math307/

HomeWork - WebWork not WebAssign

https://courses1.webwork.maa.org/webwork2/uw-math307ab/

Access etext through Canvas link.

https://canvas.uw.edu/courses/1356495/modules/items/10020079

This is the only way I will use Canvas.

Grades will be posted on Catalyst

https://catalyst.uw.edu/

Differential Equations

1. Modelling (i.e. Word Problems $F = ma$)
2. Calculus ← Easiest Part
3. Algebra
4. Interpretation

First Order Differential Equation

is a formula for the derivative of a function $\frac{dy}{dt}$ in terms of y and t.

Examples

$\frac{dy}{dt} = y + t$ or $\frac{dy}{dx} = y + x$

$\frac{dy}{dt} = \sin t$ $\frac{dy}{dx} = \sin x$

$\frac{dy}{dt} = y$ $\frac{dy}{dx} = y$

Problem Suppose an small heavy object is thrown upwards with an initial velocity of 44.7 meters/sec.

(a) write a differential equation for the velocity as a function of time [before it hits the ground] [neglecting air resistance]

(b) write the Initial Value Problem.

Solution Newton - "Force = mass · acceleration"

acceleration = derivative of velocity

$$m\frac{dv}{dt} = \text{gravitational force}$$

$$m\dot{v} = -mg \; ; \quad g = -9.8 \text{ m/s}$$

(a) DE is $\frac{dv}{dt} = -9.8$

(b) IVP is DE plus Initial Condition

$$\frac{dv}{dt} = -9.8 \qquad v(0) = 44.7 \text{ m/s}$$

Problem part (c) - solve the IVP

Solution $v(t) = -9.8t + C$

$44.7 = v(0) = -9.8 \cdot 0 + C$

$v(t) = 44.7 - 9.8t$

A newly constructed fish pond contains 2000 liters of water. Unfortunately the pond has been contaminated with 5 kg of a toxic chemical during the construction process. The pond's filtering system removes water from the pond at a rate of 200 liters/minute, removes 40% of the chemical, and returns the same volume of water to the pond. Write a differential equation for the time (measured in minutes) evolution of:

The total mass (in kilograms) of the chemical in the pond:

$\frac{dm}{dt} = ?$

I want $\frac{kg}{min}$ removed

I know 200 $\frac{liters}{min}$ pass thru filter

How many kg in each of those liter?
(this depends on m)

$\frac{m}{2000}$ kg/liter

How many of thole kg's are removed each minute?

$0.4 \cdot \frac{m}{2000} \cdot 200$

0.4: unitless; $\frac{m}{2000}$: kg/liter; 200: liters/minute

$$\frac{dm}{dt} = -\frac{0.4 \cdot 200}{2000} m$$

The concentration (in kg/liter) of the chemical in the pond:

write c in terms of m

$$c = \frac{m}{2000}$$

Relate $\frac{dc}{dt}$ to $\frac{dm}{dt}$

$$\frac{dc}{dt} = \frac{1}{2000}\frac{dm}{dt} = \frac{-1}{2000}\,0.04\,m$$

Now eliminate m from the equation.

$$\frac{dc}{dt} = -0.04\left(\frac{m}{2000}\right)$$

$$\boxed{\frac{dc}{dt} = -0.04\,c}$$

Let s = time in hours, write a DE for the concentration as a function of time in hours

$$\boxed{\frac{dc}{dt} = -0.04\,c}$$

Let s = time in hours, write a DE for the concentration as a function of time in hours

Solution - Relate s and t

$t = 60s$ minutes = hours $\cdot 60$

$$\frac{dc}{ds} = \frac{dc}{dt} \cdot \frac{dt}{ds} = (-0.04c) \cdot 60$$

$$\boxed{\frac{dc}{ds} = -2.4c}$$