How do you force a mass spring system?

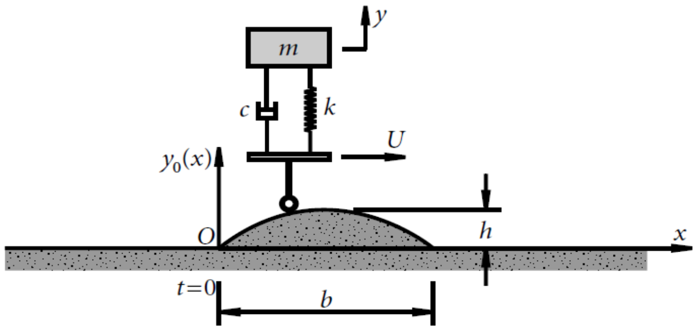
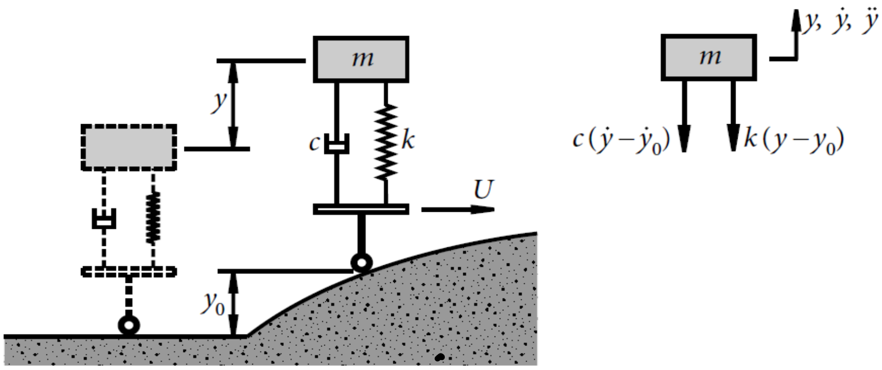


Figure 5.20 A vehicle passing a speed bump.



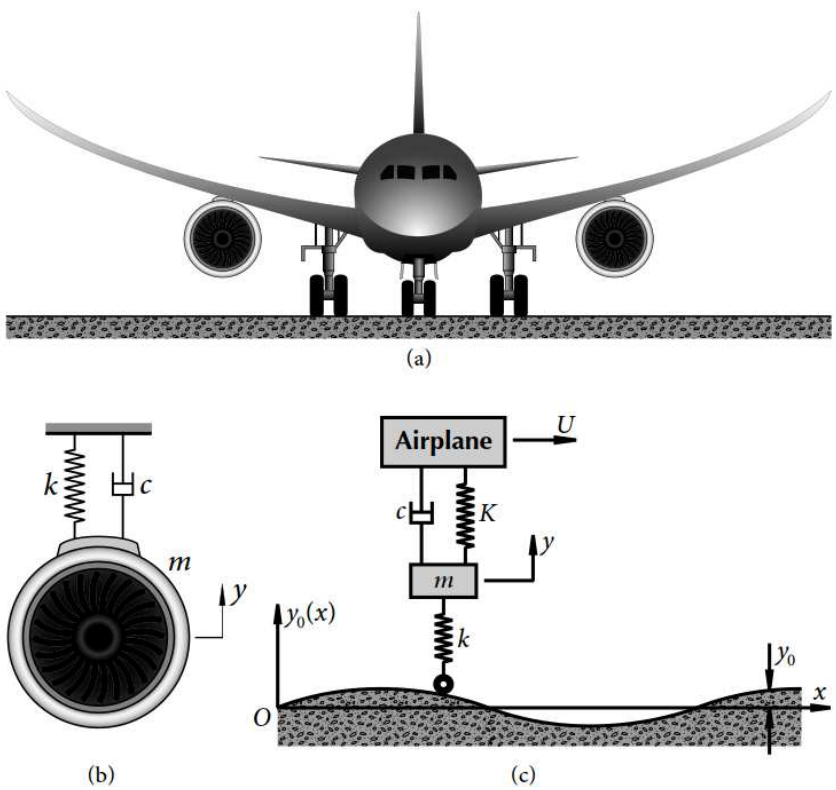


Figure 5.8 Mathematical modeling of jet engine and landing gear.

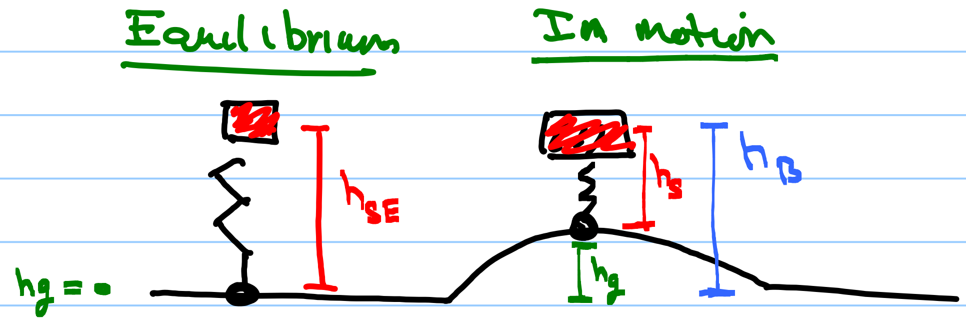
The forcing comes from driving over an uneven surface, or from a moving surface, as in an earthquake.

The force is transmitted through a spring (or a damper or both).

Height of the ground

e.g. $H_g(x) = A\cos(\alpha x)$

Mass Spring moving along bumpy ground at speed c



h_{SE} = equilibrium length of (loaded spring) = constant

$h_g(t)$ = height of the ground

$h_B(t)$ = height of the block (above $h_g = 0$ level)

Newton

$$m\,(\text{block acceleration}) = \sum \text{Forces}$$

$$m\,(\text{block height})^{\cdot\cdot} = -k\,(\text{distance spring stretched})$$

$$m\,\ddot{h}_B = -k\,(h_S - h_{SE}) = -k\,(h_B - h_g(t) - h_{SE})$$

$$m\,\ddot{h}_B + k\,h_B = k\,h_g(t) + k\,h_{SE}$$

We often set $y = h_B - h_{SE}$ (recall h_{SE} = constant)

$$m\ddot{y} + ky = k\,h_g(t)$$

$$m\ddot{y} + ky = k\,h_g(t)$$

The block (airplane/auto) is moving over the ground at speed c, i.e. $x = ct$, so

$$h_g(t) = H_g(ct)$$

where $H_g(x)$ = height of the ground at position x

If, for example, $H_g(x) = A\cos(\alpha x)$

$$m\ddot{y} + ky = kA\cos(\alpha c t)$$

<u>Notice</u>: Forcing frequency is αc, which depends on speed c and "wavenumber" α.

Bigger α means more bumps/meter.

We don't want: $\alpha c = \sqrt{\frac{k}{m}}$. Why?