Method of Undetermined Coefficients

General Procedure - Final Version

Step 2 has an additional part

① Find general homogeneous solution

(2A) Seek particular solution as a linear combination of the forcing term and its derivatives

(2B) Ask homogeneous question: Are any terms of y_p solutions to the homogeneous equation?

No → Proceed to step ③

Yes → Multiply those terms by t and repeat step 2B

---

③ Insert y_p in (DE) and solve for coefficients

Problem Find a particular solution to

$$\ddot{y}_p + y_p = \cos t$$

① Find general homogeneous solution to $\ddot{y}_H + y_H = 0$

Answer $y_H = c_1 \cos t + c_2 \sin t$

(2A) Seek particular solution as a linear combination of the forcing term and its derivatives

Answer $y_p = A \underbrace{\cos t}_{\text{Forcing term}} + B \underbrace{\sin t}_{\text{derivative of forcing term}}$

(2B) Ask homogeneous questions. Do any y_p terms match y_H terms?

Answer - yes both $\cos t$ and $\sin t$ match *

Revise $y_p = At\cos t + Bt\sin t$

Repeat (2B) Ask homogeneous questions. Do any y_p terms match y_H terms?

Answer No $t\cos t$ or $t\sin t$ doesn't match $\cos t$ or $\sin t$

Proceed to step ③

* In all the examples we looked at before, answer was No.

Step ③ $\quad \ddot{y}_p + y_p = \cos t$

$$y_p = At\cos t + Bt\sin t$$

$$\dot{y}_p = A\cos t - At\sin t + B\sin t + Bt\cos t$$

$$\ddot{y}_p = -A\sin t - A\sin t - At\cos t + B\cos t + B\cos t - Bt\sin t$$

Organize terms in $\ddot{y}_p$

$$\ddot{y}_p = -At\cos t - Bt\sin t - 2A\sin t + 2B\cos t$$

Insert in the DE

$$\ddot{y}_p = -At\cos t - Bt\sin t - 2A\sin t + 2B\cos t$$

$$+\, y_p = At\cos t \quad Bt\sin t$$

$$\cos t = 2A\sin t + 2B\cos t$$

$$A = 0 \text{ and } B = \tfrac{1}{2}$$

$$\boxed{y_p = \tfrac{1}{2} t\sin(t)}$$

General Solution $\quad y = \frac{1}{2}t\sin t + c_1\cos t + c_2\sin t$

Another Example

Find the correct form for y_p

$$\ddot{y} + 5\dot{y} + 6y = t^2 e^{-2t}$$

① $y_h = C_1 e^{-2t} + C_2 e^{-3t}$

2A Seek y_p as sum of forcing term and derivatives

$$y_p = A t^2 e^{-2t} + B t e^{-2t} + C e^{-2t}$$

2B Homogeneous Question - Yes

Multiply y_p by t

$$y_p = A t^3 e^{-2t} + B t^2 e^{-2t} + C t e^{-2t}$$

2B Repeat Homogeneous Question - No

Proceed to step ③

But the question only asked for the correct form, so the answer is:

$$y_p = A t^3 e^{-2t} + B t^2 e^{-2t} + C t e^{-2t}$$