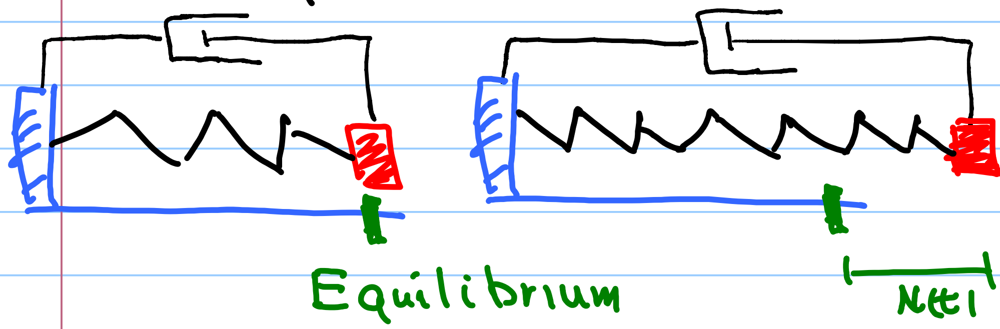
L18DampedHarmonicOscillator

Note Title 8/4/2020

Damped Harmonic Oscillator 2



Suppose the spring constant is 145 kg sec^{-2}

Damping coeff. = 2 kg sec^{-1}. $m = 1$ kg

Initial displacement = 1 meters

Initial velocity = 2 meters/sec

① Write the Initial Value Problem

② Find a formula for $x(t)$

Answer

$$m\ddot{x} = -r\dot{x} - kx$$

$$1\ddot{x} = -2\dot{x} - 145x$$

$$\ddot{x} + 2\dot{x} + 145x = 0 \qquad \text{(IVP)}$$

$$x(0) = 1 \qquad \dot{x}(0) = 2$$

$$\ddot{x} + 2\dot{x} + 145x = 0$$

$$x(0) = 1 \qquad \dot{x}(0) = 2$$

(IVP)

Seek $\quad x(t) = e^{rt}$

$$r^2 e^{rt} + 2r e^{rt} + 145 e^{rt} = 0$$

$$r^2 + 2r + 145 = 0$$

$$r^2 + 2r + 1 = -144$$

$$(r+1)^2 = -144$$

$$r + 1 = \pm 12i$$

$$r = -1 \pm 12i$$

We could write

$$x(t) = c_1 e^{(-1+12i)t} + c_2 e^{(-1-12i)t}$$

But, instead we write

$$x(t) = D_1 e^{-t} \cos 12t + D_2 e^{-t} \sin 12t$$

$$x(t) = D_1 e^{-t} \cos 12t + D_2 e^{-t} \sin 12t$$

and require $\quad x(0) = 1 \quad \dot{x}(0) = 2$

$$1 = x(0) = D_1 + \cancel{D_2 \cdot 0}$$

$$2 = \dot{x}(0) = D_1 \left(-e^{-t}\cos 12t - 12 e^{-t} \sin 12t\right)\Big|_{t=0} + D_2\left(-e^{-t}\sin 12t + 12 e^{-t}\cos 12t\right)\Big|_{t=0}$$

$$2 = D_1(-1 - 0) + D_2(0 + 12)$$

so $\quad 1 = D_1$

and $\quad 2 = -D_1 + 12 D_2$

so $\quad D_2 = \frac{3}{12} = \frac{1}{4}$

$$\boxed{x(t) = 1 e^{-t} \cos 12t + \frac{1}{4} e^{-t} \sin 12t}$$

$$x(t) = 1\,e^{-t}\cos 12t + \frac{1}{4}\,e^{-t}\sin 12t$$

$$x(t) = e^{-t}\left(1\cos 12t + \frac{1}{4}\sin 12t\right)$$

$$A = \sqrt{1+\left(\frac{1}{4}\right)^2} \qquad \phi = \operatorname{atan}\left(\frac{1/4}{1}\right)$$

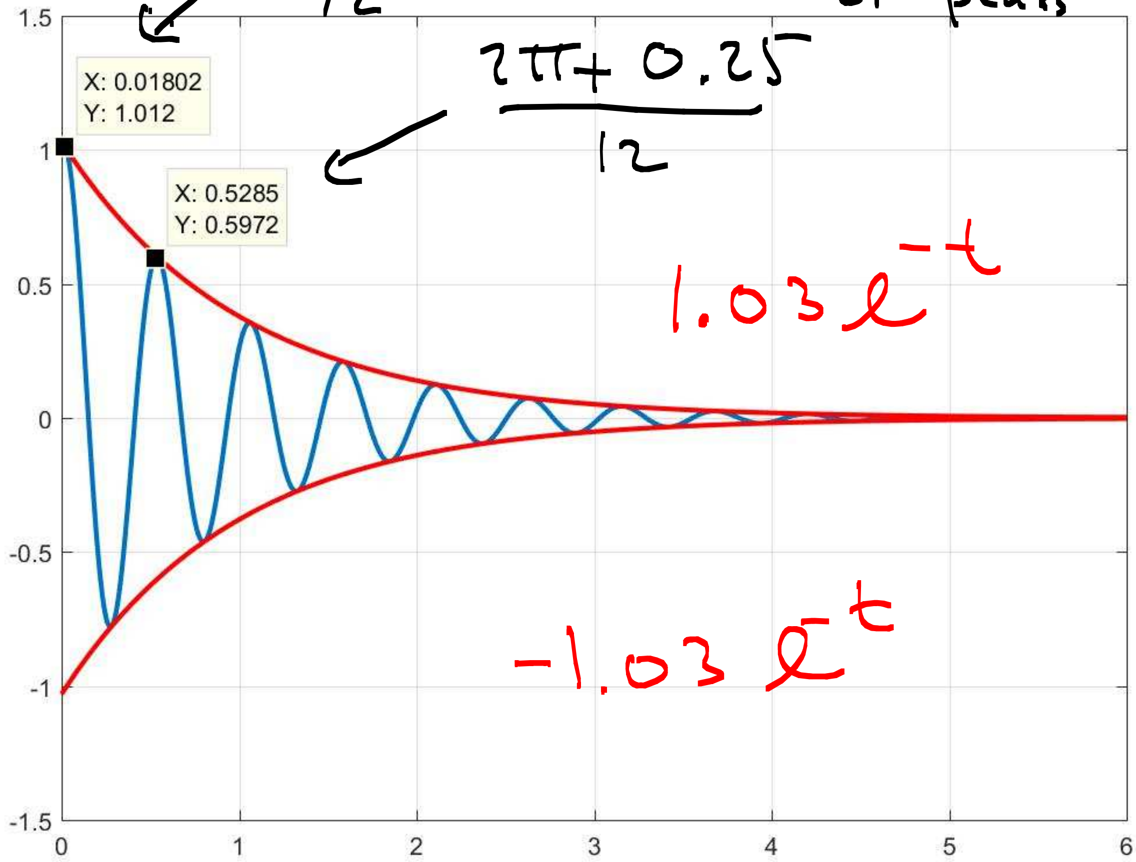
$$A = 1.03 \qquad \phi = 0.25$$

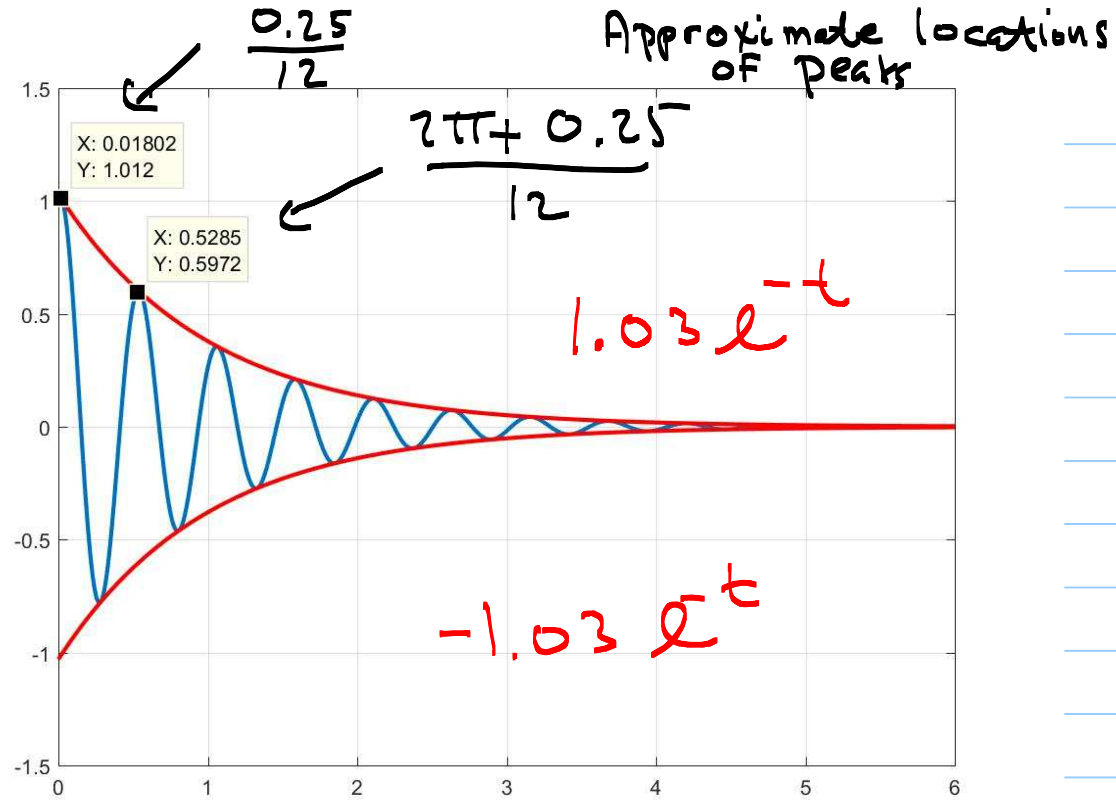
$$x(t) = 1.03\,e^{-t}\cos(12t - 0.25)$$

$\frac{0.25}{12}$

Approximate locations of peaks

$\frac{2\pi + 0.25}{12}$





$$\ddot{x} + 2\dot{x} + 145x = 0$$

$$x(t) = 1.03\, e^{-t} \cos(12t - 0.25)$$

Some Facts

The peaks occur approximately, but not exactly, at the same times as the peaks of $\cos(12t - 0.25)$.

The time between the peaks is exactly the same as the time between the peaks of $\cos(12t - 0.25)$.

12 is called the quasi-frequency and

$\frac{2\pi}{12}$ is called the quasi-period.