L12SecondOrderSimpleHarmonic

Note Title 8/3/2020

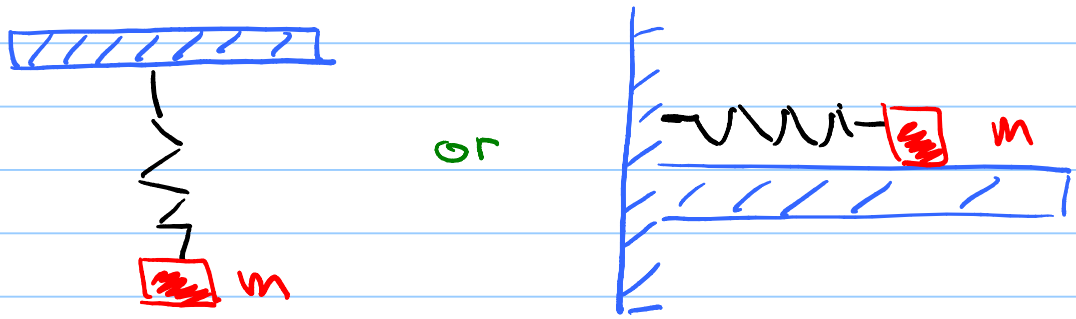
Second Order Differential Equations

Force = Mass · Acceleration

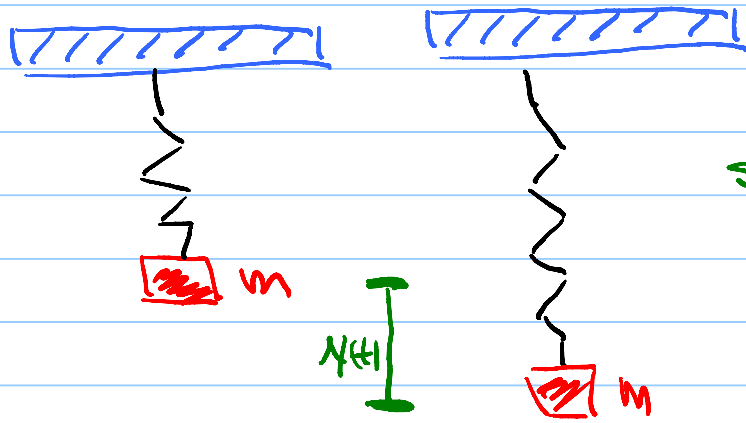
$= m \cdot \ddot{x}$

Primary Example Mass and Spring

a.k.a Simple Harmonic Oscillator



mass at rest



spring is stretched

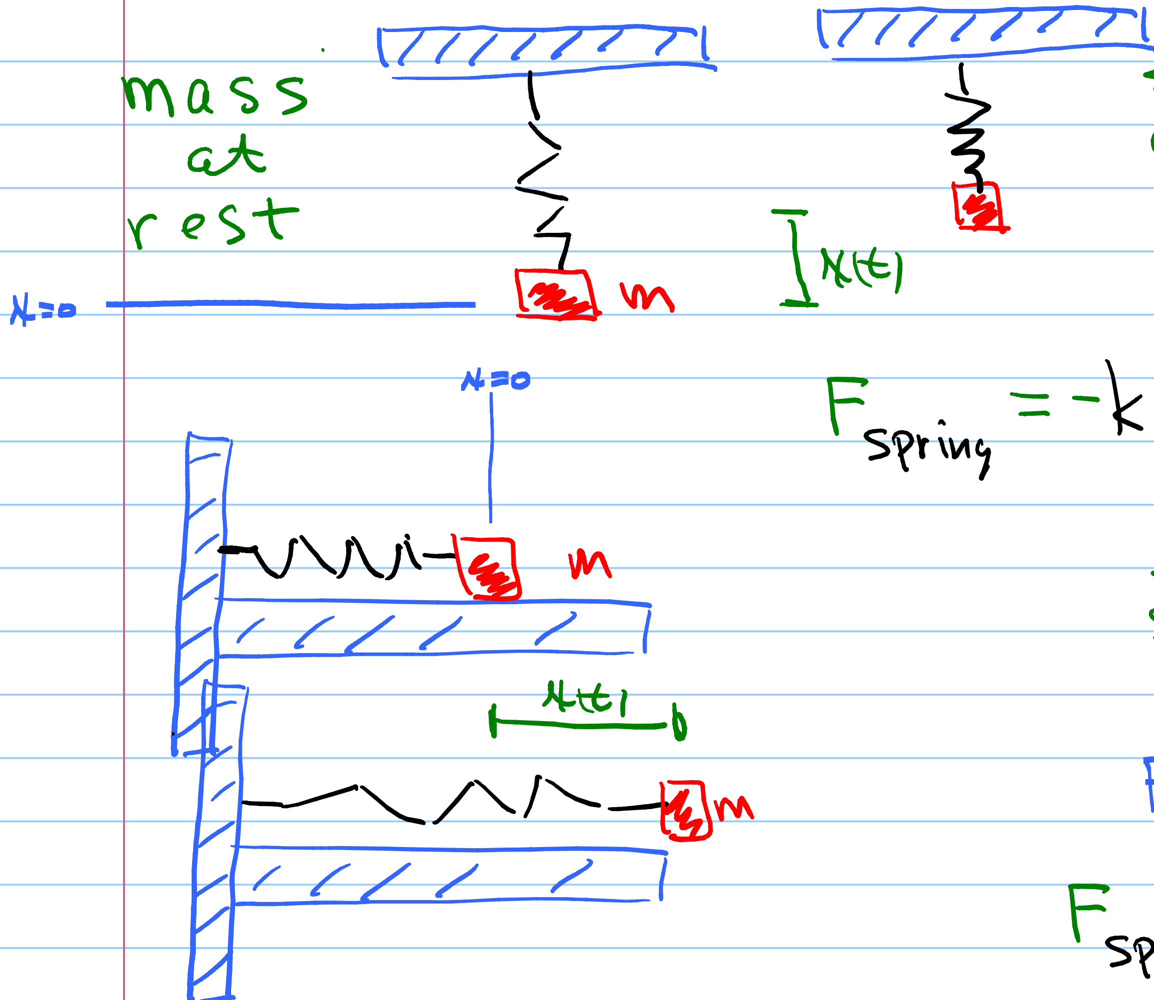
Hook's Law

$$F_{spring} = -kx$$

Spring force opposes displacement.

You may choose up or down to be the positive direction. Our text book chooses down as positive - positive displacent x when spring is stretched.

Hook's Law – Spring force is proportional to, and in the opposite direction of the displacement.



spring is compressed

$x < 0$

$F_{spring} > 0$

$$F_{spring} = -k\,x$$

spring is stretched

$x > 0$

$F_{spring} < 0$

$$F_{spring} = -k\,x$$

Equation of Motion

$$m\ddot{x} = -k\,x$$

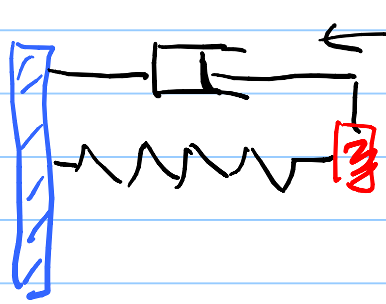
k is called the spring constant

Engineering vocabulary stiffness

https://en.wikipedia.org/wiki/Stiffness

where did gravity go? Discussion Later

Damped Harmonic Oscillator



Damper

Resists motion with a force proportional to velocity

γ = damping coefficient

https://en.wikipedia.org/wiki/Damping

piston moving through a fluid

Engineering Vocabulary dashpot

damping is different than dry friction, but acts similarly.

examples of damping

air resistance

car shock absorber

$$F_{damping} = -\gamma \dot{x}$$

Equations of Motion for Damped Harmonic Oscillator

Damping opposes velocity

$$m\ddot{x} = -\gamma \dot{x} - kx$$

Spring force opposes displacement

Usually written

$$m\ddot{x} + \gamma \dot{x} + kx = 0$$

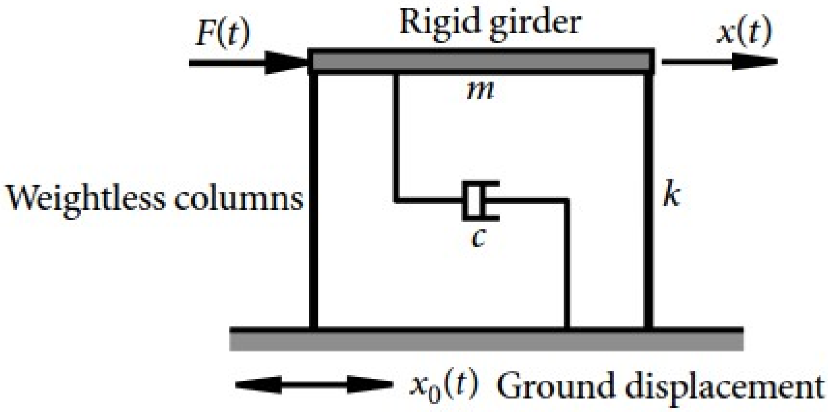
Damped Harmonic Oscillators are Ubiquitous (everywhere)

*

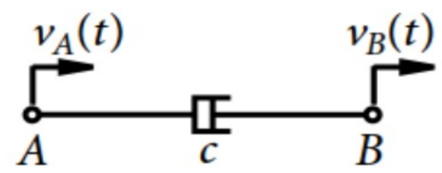
Modelling Building Response to an Earthquake

A single story shear building consists of a rigid girder with mass m, which is supported by columns with combined stiffness k. The columns are assumed to be weightless, inextensible in the axial (vertical) direction, and they can only take shear forces but not bending moments. In the horizontal direction, the columns act as a spring of stiffness k. As a result, the girder can only move in the horizontal direction, and its motion can be described by a single variable $x(t)$; hence the system is called a single degree-of-freedom (DOF) system. The number of degrees-of-freedom is the total number of variables required to describe the motion of a system.

$$m\ddot{x} = -kx - c\dot{x}$$



The internal friction between the girder and the columns is described by a viscous dashpot damper with damping coefficient c. A dashpot damper is shown schematically in Figure 5.3 and provides a damping force $-c(v_B - v_A)$, where v_A and v_B are the velocities of points A and B, respectively, and $(v_B - v_A)$ is the relative velocity between points B and A. The damping force is opposite to the direction of the relative velocity.



* I don't expect you to understand all the words here. I simply want to give you the (correct) impression that the damped harmonic oscillator is one of the most important models in science and engineering.

The combined stiffness k of the columns can be determined as follows. Apply a horizontal static force P on the girder. If the displacement of the girder is Δ as shown in Figure 5.2, then the combined stiffness of the columns is $k = P/\Delta$.

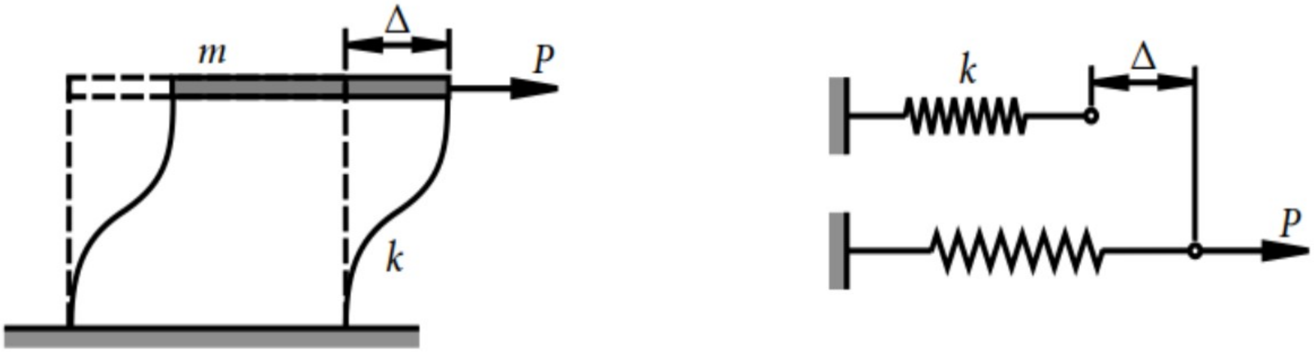
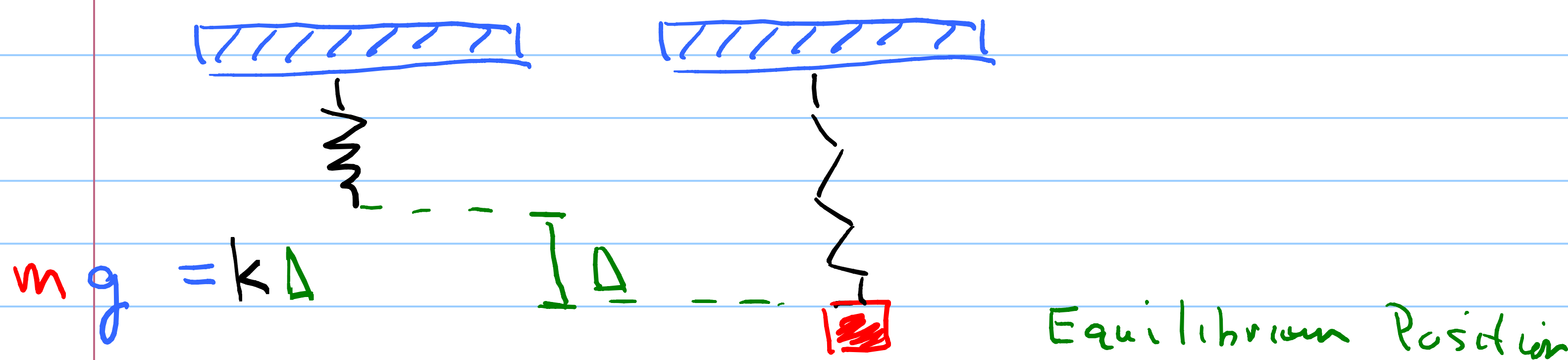


Figure 5.2 Determination of column stiffness.

You will be expected to determine k for a mass spring system from "m" and Δ.

Determining k for mass-spring system (damped or undamped)



At equilibrium, sum of forces is zero

$0 = mg - k\Delta$

$$\boxed{k = \frac{mg}{\Delta}}$$

More Propaganda For Mass Spring Systems

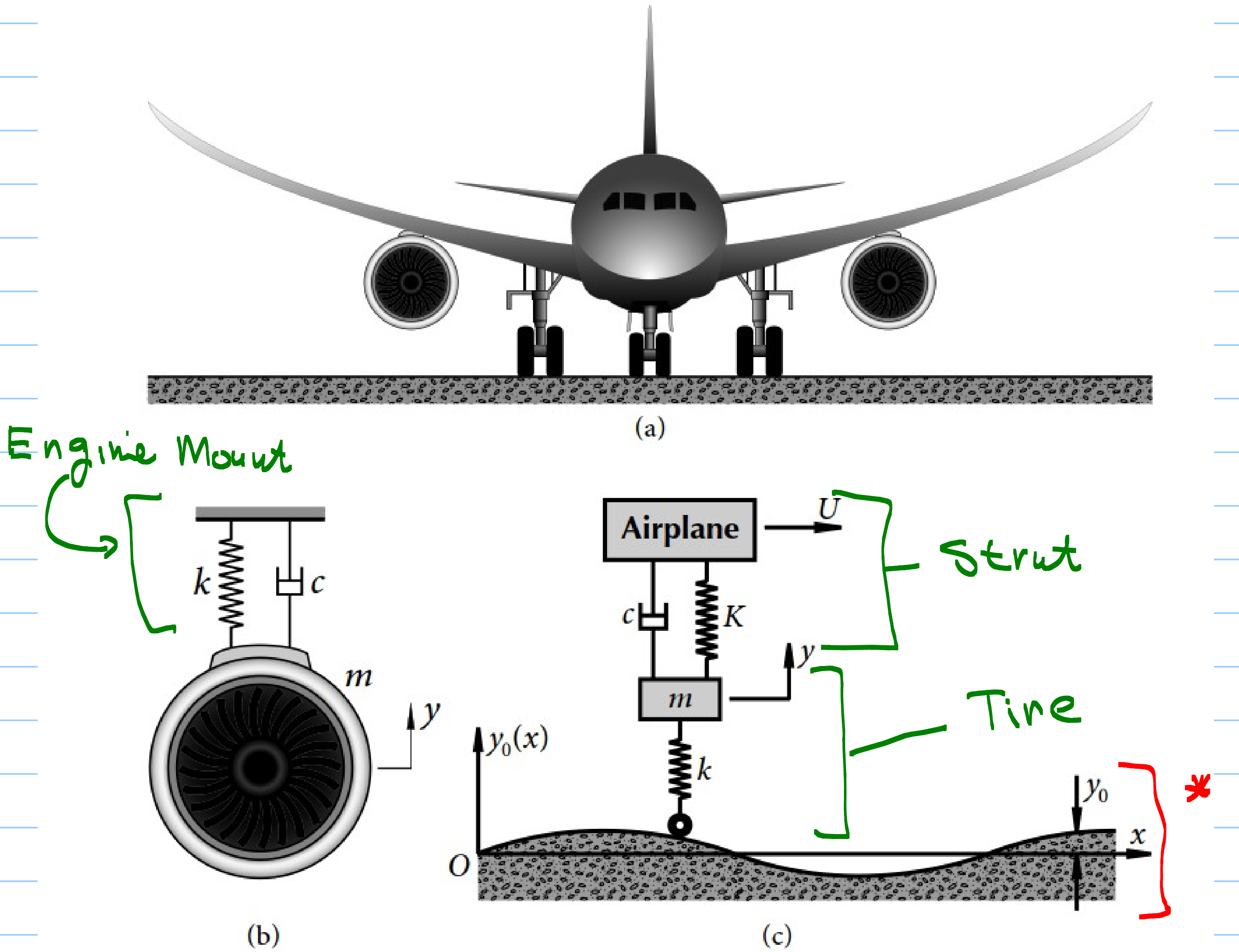
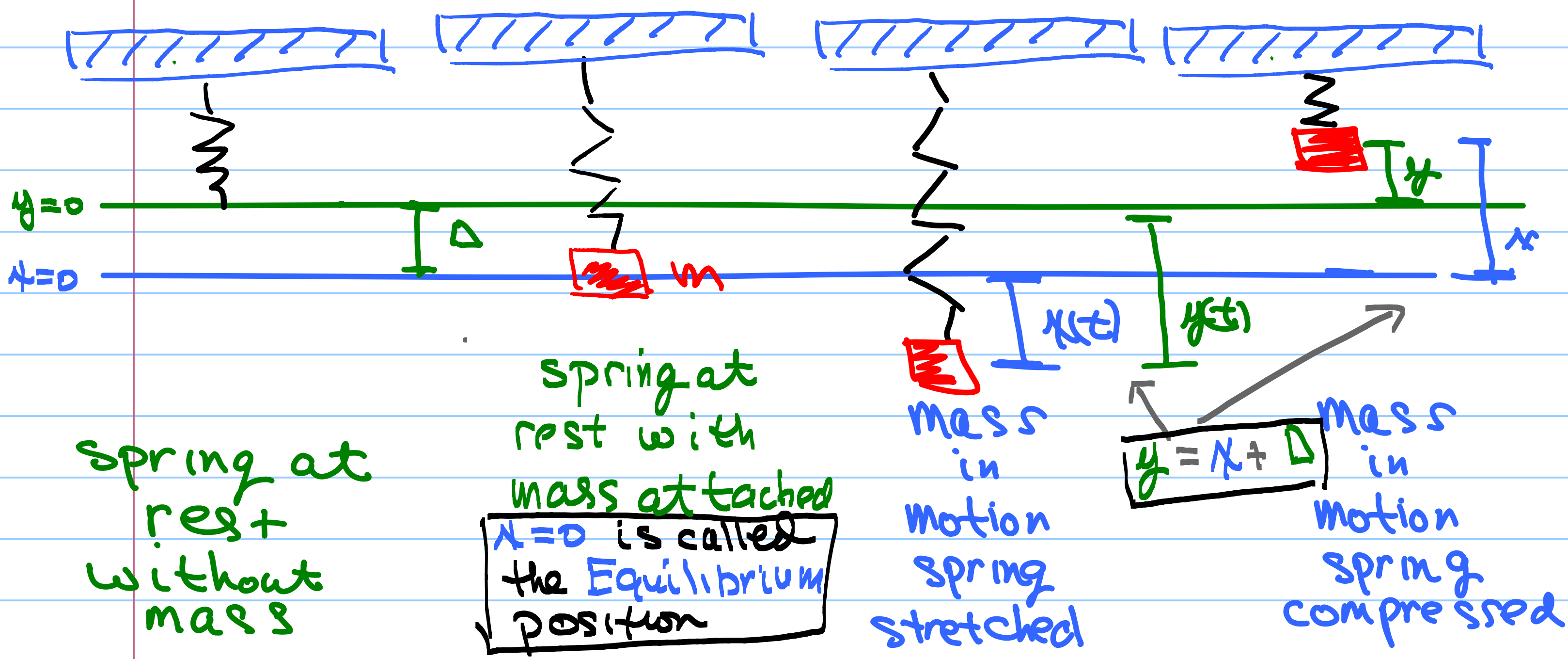


Figure 5.8 Mathematical modeling of jet engine and landing gear.

* As the airplane rolls over uneven ground, the tire and the airplane vibrate (oscillate). The system design problem is to choose c, K, m, k so that these oscillations don't get too big.

where did gravity go?*



<u>Mass at Rest</u> - Net Force = 0

$0 = -k\Delta + mg$ = Spring Force + Gravitational Force

DE for $y(t)$ $\qquad m\ddot{y} = F_{spring} + F_{gravity}$

$$m\ddot{y} = -ky + mg$$

But Everyone writes DE for the dispacent from equilibrium $\qquad x(t) = y - \Delta$

$$m\ddot{x} = m\ddot{y} = -ky + mg = -k(x+\Delta) + mg$$

$$= -kx \quad \underbrace{-k\Delta + mg}_{=0}$$

$$\boxed{m\ddot{x} = -kx}$$

* I won't ask you to reproduce this derivation.