

Equilibria and Stability

Problem

$$DE \quad \dot{y} = y(y-6)(y-4)$$

- ① Find the equilibrium solutions
- ② Sketch the direction field
- ③ Sketch the phase line
- ④ Label equilibrium solutions as stable or unstable

Equilibrium Solutions are $y = \text{constant}$

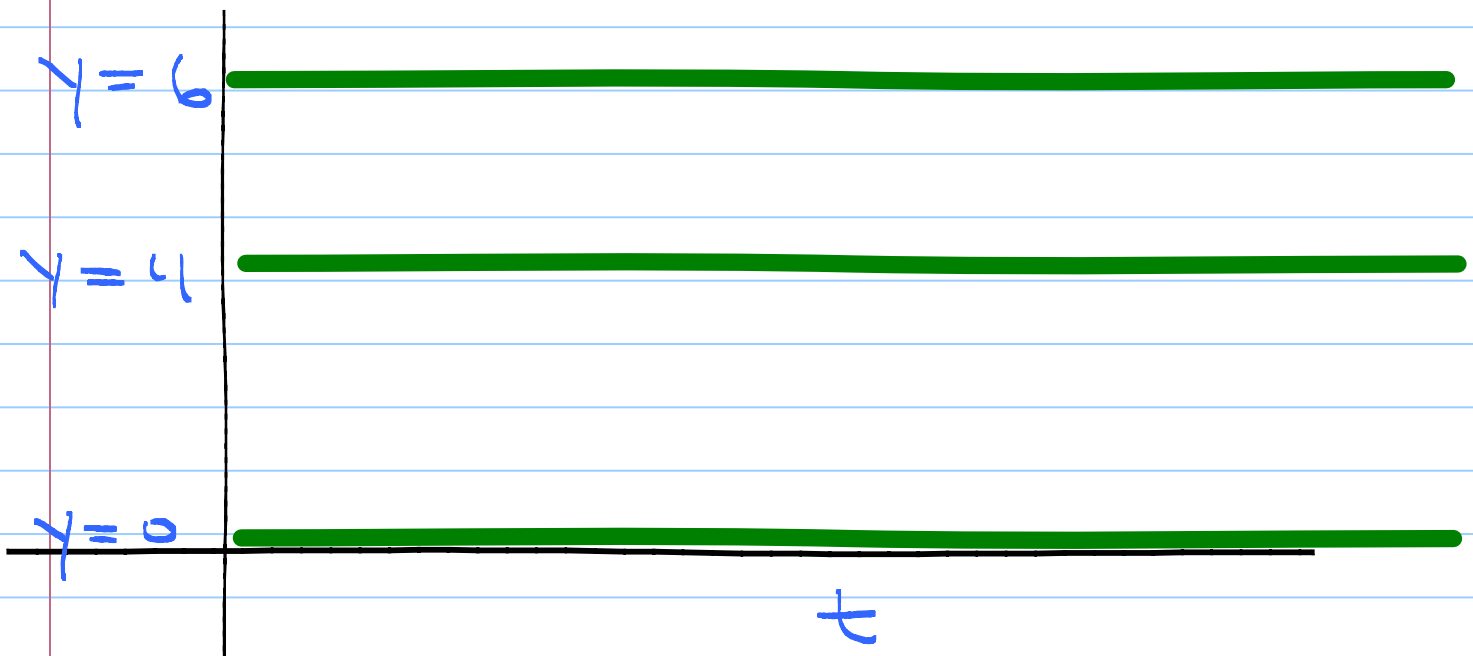
If $y = \text{constant}$, $\dot{y} = 0$, so we must

$$0 = \dot{y} = y(y-4)(y-6)$$

So equilibrium solutions are

$$y = 0 ; y = 4 ; y = 6$$

Draw them:



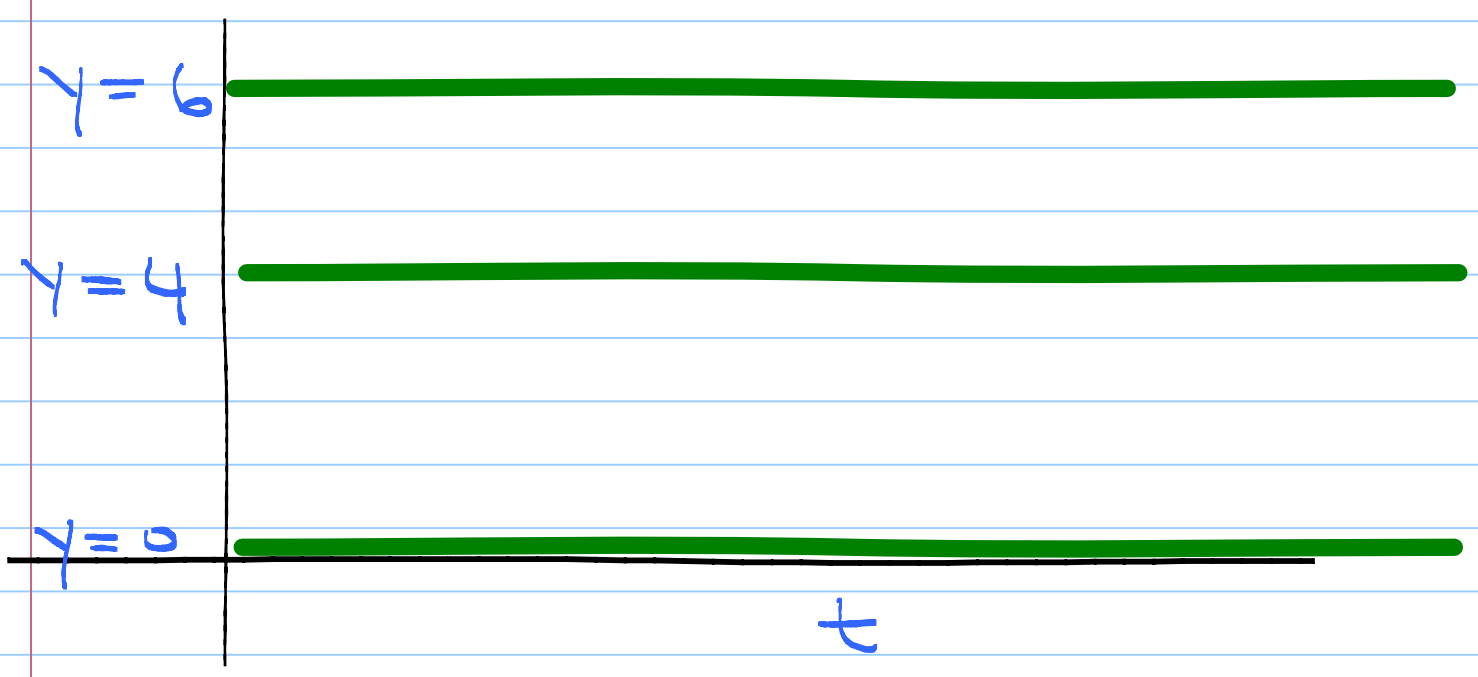
Step 1 Sketch Dfield

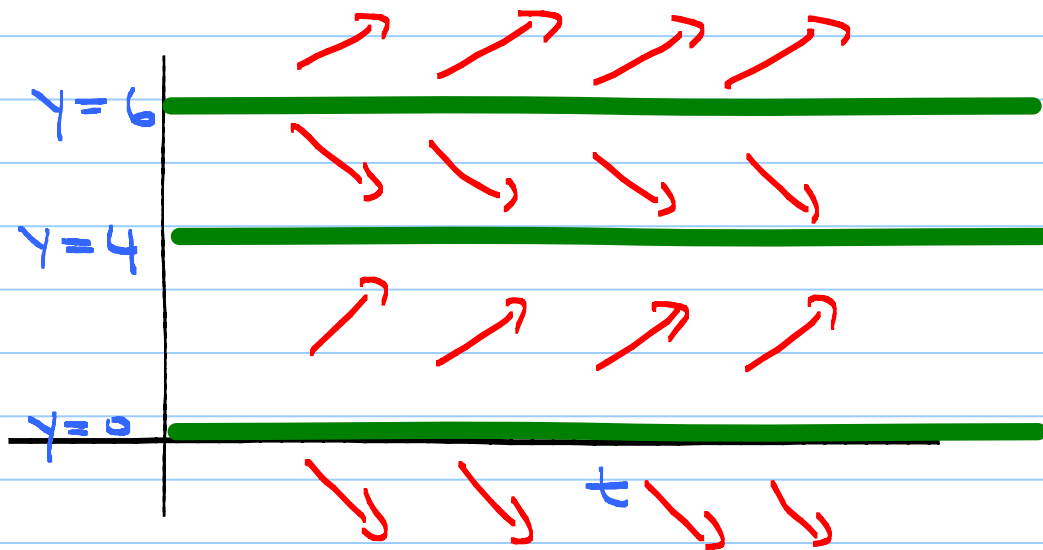
HP $y < 0$ $\dot{y} = y(y-6)(y-4) < 0$

HP $0 < y < 4$ $\dot{y} = y(y-6)(y-4) > 0$

HP $4 < y < 6$ $\dot{y} = y(y-6)(y-4) < 0$

HP $6 < y$ $\dot{y} = y(y-6)(y-4) > 0$

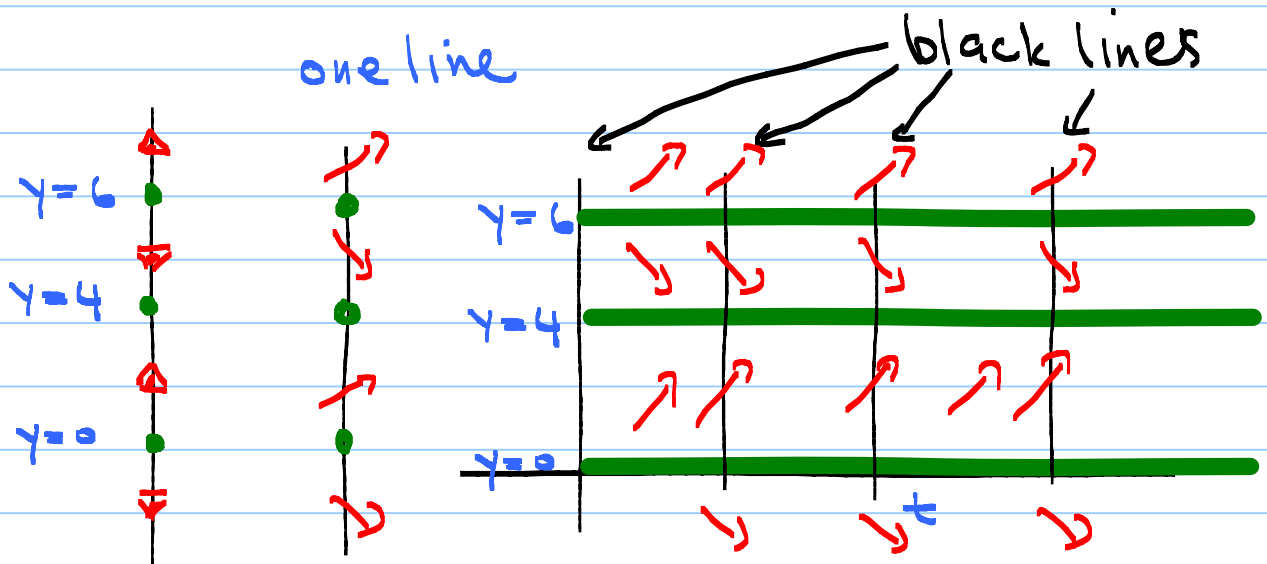




Sketch the Phase Line

Notice that $F(y)$ doesn't involve t .

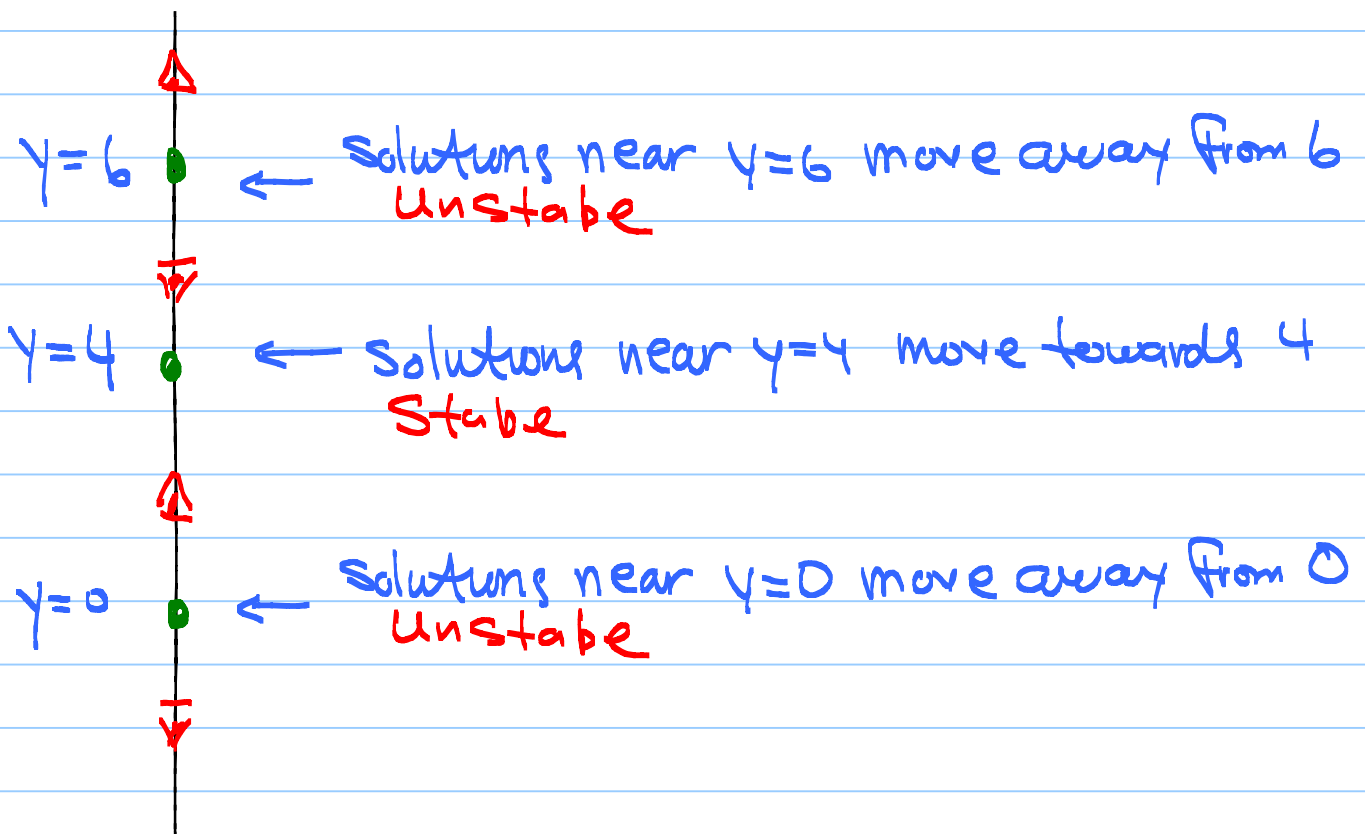
The Direction Field looks the same on each of the black lines below so we can make one line that summarizes all the DField information.



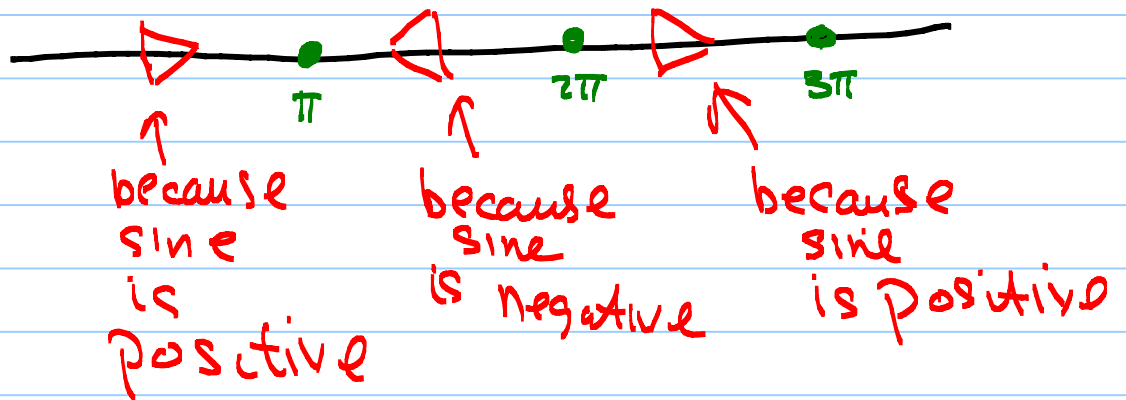
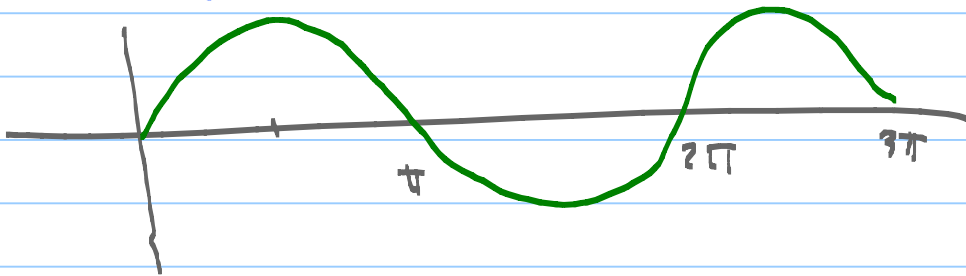
↖ The Phase line

But we usually draw it differently

Label equilibrium solutions as
stable or unstable



Problem $y' = \sin y$ Draw the Phase Line



Now turn it sideways

