L07FOLNewtonCooling

Note Title 7/28/2020

Integrating Factor Summary

① Linear First Order DE

$$\frac{dy}{dt} + p(t)y = q(t)$$

② Find $m(t)$ by solving $\frac{d}{dt} m = p\,m$

we can write a formula $m = e^{\int p(t)}$

③ Multiply First Order DE by m

$m\frac{dy}{dt} + m p(t) y$ equals $\frac{d}{dt}(m y)$

④ Integrate both sides of

$$\frac{d}{dt}(m y) = m\,q(t)$$

and divide by m

$$y(t) = \frac{1}{m}\left(\int m\,q(t)\right) + \frac{c}{m}$$

to obtain the general solution.

I don't remember most of these Formulas.
I just remember that I want

$m\frac{dy}{dt} + m p(t) y$ to be $\frac{d}{dt}(m y) = m\frac{dy}{dt} + \frac{dm}{dt} y$

so $\frac{dm}{dt} = mp$

Example 2 Newton's Law of Cooling

Temperature T of an object changes at a rate proportional to the deviation from ambient temperature. Suppose time is in minutes, ambient temperature is $(20 + 5e^{-.02t})^\circ C$, the constant of proportionality is $0.01\ min^{-1}$, and the initial temperature of the object is $40^\circ C$.

(a) Write the (IVP).

(b) Solve the (IVP)

(IVP) $\frac{dT}{dt} = 0.01\left(20+5e^{-0.02t} - T\right)$

$T(0) = 40$

Why not

$\left(T - \left(20+5e^{-0.02t}\right)\right)$?

Solve

$$\frac{dT}{dt} + 0.01\,T = 0.2 + 0.05\,e^{-0.02t}$$

Find integrating factor

Multiply $\frac{dT}{dt} + 0.01T$ by something so that it becomes the derivative of a product.

Answer $e^{0.01t}$

$$e^{0.01t}\frac{dT}{dt} + 0.01\,e^{0.01t}T = \frac{d}{dt}\left(e^{0.01t}T\right)$$

$$\frac{dT}{dt} + 0.01T = 0.2 + 0.05e^{-0.02t}$$

Multiply by $e^{0.01t}$

$$e^{0.01t}\frac{dT}{dt} + 0.01e^{0.01t}T = 0.2e^{0.01t} + 0.05e^{-0.01t}$$

Recognize product rule

$$\frac{d}{dt}\left(e^{0.01t}T\right) = 0.2e^{0.01t} + 0.05e^{-0.01t}$$

Integrate $\int e^{at} = \frac{e^{at}}{a}$

$$e^{0.01t}T = \frac{0.2}{0.01}e^{0.01t} - \frac{0.05}{0.01}e^{-0.01t} + C$$

Multiply by $e^{-0.01t}$

$$T(t) = 20 - 5e^{-0.02t} + Ce^{-0.01t}$$

This is the general solution

Find C $\quad 40 = T(0) = 20 - 5 + C$

$$25 = C$$

$$T(t) = 20 - 5e^{-0.02t} + 25e^{-0.01t}$$

This is the solution to the IVP