Hefei lectures: Weighted projective lines and applications

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My Hefei lectures will present various aspects of weighted projective lines, including applications. The subject will be arranged in six talks, whose (tentative) descriptions are given below.

Lecture 1: Definition and basic properties

First, I will deal with historical aspects, motivation and various perspectives to look at weighted projective lines. Then I will follow the 1987-paper bei Geigle-Lenzing to define the categories of coherent sheaves on a weighted projective line, and establish their basic properties including Serre duality and the existence of the canonical tilting object.

Lecture 2: The role of the Euler characteristic

This lecture will be devoted to the important role of the Euler characteristic of a weighted projective which has an easy definition in terms of the weights, but has important implications. In particular, the complexity of the classification of indecomposables has a completely different flavor if the Euler characteristic is positive, zero or negative, respectively. We will discuss in detail the complete classification for positive Euler characteristic. In this lecture we will also deal with the technical aspects of stability and semi-stability.

Lecture 3: Classification aspects for zero or negative Euler characteristic

Here, we sketch from joint work with Hagen Meltzer the classification of indecomposable sheaves for Euler characteristic zero (tubular case), and explain the implications for the representation theory of tubular algebras. From joint work with José-Antonio de la Peña we also summarize what is known for negative Euler characteristic when the classification problem is wild.

Lecture 4: Vector bundles and (graded) Cohen-Macaulay modules

Here, we speak on the link between weighted projective lines $\mathbb{X}$ and commutative algebra. We will discuss the influence of the graded factoriality of their coordinate algebras and, more importantly, the relationship to singularity theory through the correspondence between vector bundles on $\mathbb{X}$ and graded Cohen-Macaulay modules over the (graded) coordinate algebra of $\mathbb{X}$.

Lectures 5 and 6: Stable categories of vector bundles

While the results of Lectures 1 to 4 summarize results that are known for about 20 years, the last two Lectures will focuss on recent results obtained in joint work with J. A. de la Peña respectively with D. Kussin and H. Meltzer. These results increase substantially the range of applicability of hereditary representation theory. The key observation is that the category vect$\mathbb{X}$ of vector bundles on $\mathbb{X}$ is naturally equipped with the structure of a Frobenius category such that the line bundles on $\mathbb{X}$ are exactly the indecomposable projective (=injective) objects with respect to this structure.

In particular, the associated stable category $\mathcal{T} = \underline{\mathrm{vect}}\,\mathbb{X}$ is a triangulated category having nice mathematical properties. In particular, $\mathcal{T}$ has Serre duality and almost sequences; further $\mathcal{T}$ has a tilting object and is fractionally Calabi-Yau. The Calabi-Yau dimension, moreover, is a function of the Euler characteristic $\chi_{\mathbb{X}}$. The subject is linked in various ways to singularity theory since $\underline{\mathrm{vect}}\,\mathbb{X}$ has an interpretation as the stable category of $\mathbb{L}$-graded CM-modules over S.