Introduction to the weighted projective lines

(1) **Constructing weighted projective lines from canonical algebras.**
  - Rough structure of $\mathrm{mod}(\Lambda)$ (Λ canonical), separating tubular family. ([7])
  - Sketch of tubular shifts [4, (S10)].
  - Construction of $\mathcal{H} = \mathrm{coh}(\mathbb{X})$ in $\mathrm{D}^b(\Lambda)$ and its properties ([4, Thm. 6.1]). (Hereditary, noetherian, ... with tilting object T such that $\mathrm{End}(T) = \Lambda$.)

(2) **Axiomatic approach to weighted projective lines.**
  - Starting with axioms ($\mathcal{H}$ hereditary, noetherian, ... category with tilting object like in [6, Ch. 10], or [5]) show existence of tilting object T with $\mathrm{End}(T)$ canonical and that $\mathcal{H} = \mathrm{coh}(\mathbb{X})$ as in (1).
  - Review Happel's theorem classifying hereditary categories with tilting object up to derived equivalence. ([6, Thm 6.3].)

(3) **Vector bundles and Cohen-Macaulay modules.**
  - The graded algebra $S(p, \lambda)$. ([2, Ch. 1].)
  - Serre's theorem [2, 1.8]. Avoiding sheaves one can sketch the (much more general) proof in [1, Thm. 4.5] (using [2, (1.5.5)]).
  - Correspondence between torsionfree objects (vector bundles) and CM-modules [2, Thm. 5.1], [3, 7.5+8.3].

(4) **Representation type and Euler characteristic.** Stability (semistability, resp.) of indecomposable bundles in case of positive (zero, resp.) Euler characteristic. ([2, Ch. 5] and [6, Ch. 10].) Shape of the categories in case of positive, zero, negative Euler characteristic, resp.

References

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