Let $p$ and $n$ be positive integers and suppose that $p$ is prime. There is a unique (up to isomorphism) field with $q = p^n$ elements. We denote it by $\mathbb{F}_q$. For example, $\mathbb{F}_p \cong \mathbb{Z}_p = \mathbb{Z}/(p)$.

1. If $a, b, c, d \in R$ show that $(a, c) = (d, c)$.
2. In $\mathbb{Q}[x]$, find a single generator for the ideal $(x^3 - 2x + 1, x^2 - 3)$. Explain what you are doing.
3. Show that the cube root of 7 is irrational, stating carefully any theorems about $\mathbb{Z}$ that you use.
4. The polynomials in $\mathbb{Z}[x]$ having a constant term that is a multiple of 6 form an ideal. Explain why by realizing this set as the kernel of a homomorphism (state the theorem you are using), and give generators for that ideal.
5. Let $C$ be the ring of continuous functions from $\mathbb{R}$ to $\mathbb{R}$. Define a function $\psi$ from $C$ to $\mathbb{R}$ by $\psi(f) = f(13)$. State whether each of the following results is true or false.
   (a) $\psi$ is injective.
   (b) $\psi$ is surjective.
   (c) $\psi$ is a ring homomorphism.
6. Why is the ring $\mathbb{F}_{19}[x]/(x^2 + 2)$ not a field? State the result(s) you are using.
7. Find two non-zero elements in $\mathbb{F}_{19}[x]/(x^2 + 2)$ whose product is zero.
8. Explain why the ring $F = \mathbb{F}_{19}[x]/(x^2 + 5)$ is a field.
9. What is the inverse of $x + 1$ in $F$?
10. Define prime and irreducible elements in a commutative domain $R$.
11. Discuss what you know about the relationship between the notions of prime and irreducible elements.