MATH 16A WORKSHEET 3

TUE, JAN 30, 2018

(1) Find the domain of the function $f(x) = \sqrt{x^2 - 4x - 5}$.

The only condition imposed on the function $f(x)$ is that the input to a square root must be a nonnegative real number (zero is allowed). Thus $x^2 - 4x - 5 \geq 0$. Factoring the left hand side gives $(x-5)(x+1) \geq 0$. There are two cases:

Suppose $x - 5 \geq 0$ and $x + 1 \geq 0$. This is the same as saying $x \geq 5$ and $x \geq -1$, which is the same as saying just $x \geq 5$.

Suppose $x - 5 \leq 0$ and $x + 1 \leq 0$. This is the same as saying $x \leq 5$ and $x \leq -1$, which is the same as saying just $x \leq -1$.

Thus our final answer is $\boxed{(-\infty, -1] \cup [5, \infty)}$.

(2) Complete the square and determine the vertex for the equation $y = -2x^2 + 8x - 9$.

First we factor out the coefficient of x^2 to get $y = -2(x^2 - 4x + \frac{9}{2})$. Then we add and subtract (inside the parentheses) $(b/2)^2$ where b is the coefficient of x to get $y = -2(x^2 - 4x + 4 - 4 + \frac{9}{2})$ (in this case $b = -4$). Factoring the first three terms inside the parentheses and combining the other two constants gives $y = -2((x-2)^2 + \frac{1}{2})$. Distributing the -2 gives $y = -2(x-2)^2 - 1$. Thus our vertex is $(h, k) = \boxed{(2, -1)}$.

(3) Solve the equation:

$$e^x = \frac{1}{e^5}$$

We rewrite the right hand side in the form $e^{\text{something}}$ to get $e^x = e^{-5}$. Thus $x = \boxed{-5}$.

(4) Solve the equation:

$$8^{x^2} = 2^{x+4}$$

We rewrite the left hand side as a power of 2 to get $(2^3)^{x^2} = 2^{x+4}$. Simplifying the exponents in the left hand side gives $2^{3x^2} = 2^{x+4}$. Comparing exponents on both sides gives $3x^2 = x + 4$, and moving everything to one side gives $3x^2 - x - 4 = 0$, and factoring gives $(3x-4)(x+1) = 0$. Thus $x = \boxed{\frac{4}{3}, -1}$.