MATH 1A QUIZ 1

Please write your solutions on a separate sheet of paper. Be sure to write your name and section number at the top of each page.

Problem 1. (15 points)
- (i) State the Squeeze Theorem.
- (ii) Prove the Squeeze Theorem.
- (iii) Use the Squeeze Theorem to find

$$\lim_{x \to 0} \frac{x^4}{10} \cos \frac{2\pi}{5x} .$$

 Justify your answer carefully.

Problem 2. (30 points)
- (i) State the definition of limit for sequences (i.e. what exactly does $\lim_{n \to \infty} f(n) = L$ mean?).
- (ii) Prove that

$$\lim_{n \to \infty} \left(\frac{3}{4} \right)^n = 0 .$$

- (iii) Prove that

$$\lim_{n \to \infty} \frac{n^3 - 1}{n^3} = 1 .$$

Problem 3. (35 points)
- (i) State the definition of limit for functions (i.e. what exactly does $\lim_{x \to a} f(x) = L$ mean?).
- (ii) Let $f(x) = \sqrt{x - 3}$. Find a real number δ such that the following is true: if x is a real number such that $0 < |x - 7| < \delta$, then $|f(x) - 2| < \frac{1}{3}$.
- (iii) Prove that

$$\lim_{x \to 0} x^{43} = 0 .$$

- (iv) Prove that

$$\lim_{x \to 3} x^2 - 4x = -3 .$$

Problem 4. (10 points) Evaluate the following limits and justify each step by indicating the appropriate Limit Laws.
- (i)

$$\lim_{x \to -2} \left(\frac{t^2 - 2}{2t^2 - 3t + 2} \right)^3$$

- (ii)

$$\lim_{x \to 2} \sqrt{\frac{2x^2 + 1}{3x - 2}}$$

Problem 5. (10 points)
- (i) What exactly does it mean for a function $f(x)$ to be continuous at the point $x = a$?
- (ii) State the Intermediate Value Theorem.
- (iii) Use it to show that the polynomial $p(x) = x^2 - \pi x + 2$ has a root between 0 and 1.