FINAL REVIEW DAY 1

WED, NOV 27, 2013

Problem 1. Prove that

$$\lim_{x\to 1} \frac{x^2+x-3}{x+2} = -\frac{1}{3}$$

using the ϵ, δ definition of limit.

Problem 2. Show that the equation $x^4 - 10x^2 + 5 = 0$ has a root in the interval $(0, 2)$.

Problem 3 (pg. 265, #54). Let $f(x) = \frac{1}{2-x}$. Find a general formula for $f^{(n)}(x)$.

Problem 4 (pg. 265, #55). Let $f(x) = xe^x$. Prove that $f^{(n)}(x) = (x+n)e^x$.

Problem 5 (pg. 267, #109). Evaluate

$$\lim_{x\to 0} \frac{\sqrt{1+\tan x} - \sqrt{1+\sin x}}{x^3}.$$

Problem 6 (pg. 267, #110). Suppose f and g are differentiable functions such that $f(g(x)) = x$ and $f'(x) = 1 + (f(x))^2$. Show that $g'(x) = \frac{1}{1+x^2}$.

Problem 7 (pg. 353, #50). Find two positive integers m, n such that $m + 4n = 1000$ and mn is as large as possible.

Problem 8 (pg. 353, #52). Find the point on the hyperbola $xy = 8$ that is closest to the point $(3, 0)$.

Problem 9 (pg. 420, #3). If $\int_0^4 e^{(x-2)^4}\, dx = k$, find the value of $\int_0^4 xe^{(x-2)^4}\, dx$.

Problem 10 (pg. 420, #6). If $f(x) = \int_0^x x^2 \sin(t^2)\, dt$, find $f'(x)$.

Problem 11 (pg. 420, #9). Find the interval $[a, b]$ for which the value of the integral $\int_a^b (2 + x - x^2)\, dx$ is a maximum.

Problem 12 (pg. 436, Example 7). A solid has a circular base of radius 1. Each cross-section of the solid by a plane perpendicular to the base is a square. Compute the volume of the solid.

Problem 13. Let $\{(x, y, z) \ :\ x^2 + y^2 = 1\}$ and $\{(x, y, z) \ :\ x^2 + z^2 = 1\}$ be two cylinders of radius 1. Find the volume of the solid defined by the intersection of these cylinders.

Problem 14 (pg. 445, #12). Let R be the region bounded by the curves $y = 4x^2 - x^3$ and $y = 0$. Use cylindrical shells to find the volume of the solid obtained by rotating R about the y-axis.