INTEGRATION PRACTICE

WED, NOV 20, 2013

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Problem 1. Find an antiderivative of the following functions:

(1) $x \ln x$: integrate by parts, using $u = \ln x$ and $v' = x$.
(2) $x^2 \ln x$: integrate by parts, using $u = \ln x$ and $v' = x^2$.
(3) $\frac{\sin 2x}{\sin x}$: write $\sin 2x = 2(\sin x)(\cos x)$.
(4) $\frac{1}{\sqrt{a^2-x^2}}$: write $\frac{1}{\sqrt{a^2-x^2}} = \frac{\frac{1}{a}}{\sqrt{1-(\frac{x}{a})^2}}$ and use the substitution $u = \frac{x}{a}$; recall that $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$.
(5) $x^2 e^x$: integrate by parts, using $u = x^2$ and $v' = e^x$ (you're going to have to find $\int xe^x\, dx$, which you can find by parts again with $u = x$ and $v' = e^x$).
(6) $e^x \cos x$: let F_1 be an antiderivative of $e^x \cos x$ and F_2 an antiderivative of $e^x \sin x$. Integrate by parts (using $u = e^x$ and $v' = \cos x$) to get $F_1(x) = e^x \sin x - F_2(x) + C_1$ for some constant C_1. Integrate by parts (using $u = e^x$ and $v' = \sin x$) to get $F_2(x) = -e^x \cos x + F_1(x) + C_2$ for some constant C_2. Solving for $F_1(x)$ gives $F_1(x) = \frac{e^x \cos x + e^x \sin x}{2} + C$.
(7) $x \cos x$: integrate by parts, using $u = x$ and $v' = \cos x$.
(8) $\frac{1}{x^2-1}$: by partial fraction decomposition, write $\frac{1}{x^2-1} = \frac{1/2}{x-1} + \frac{-1/2}{x+1}$.
(9) $\frac{1}{x^2+1}$: This is just $\arctan x$. (You can also use the substitution rule backwards, letting $x = \tan u$.)
(10) $\frac{1}{x^3+1}$: by partial fraction decomposition, write

$$\frac{1}{x^3+1} = \frac{1/3}{x+1} + \frac{(-1/3)x + (2/3)}{x^2 - x + 1} .$$

Refer to the "Integrating Rational Functions" handout on how to integrate $\frac{(-1/3)x+(2/3)}{x^2-x+1}$.
(11) $\frac{1}{x^4+1}$: by partial fraction decomposition, write

$$\begin{aligned}\frac{1}{x^4+1} &= \frac{1}{(x^2 - \sqrt{2}x + 1)(x^2 + \sqrt{2}x + 1)} \\ &= \frac{\frac{-1}{2\sqrt{2}}x + \frac{1}{2}}{x^2 - \sqrt{2}x + 1} + \frac{\frac{1}{2\sqrt{2}}x + \frac{1}{2}}{x^2 + \sqrt{2}x + 1} .\end{aligned}$$

Refer to the "Integrating Rational Functions" handout on how to integrate each of the above two terms.
(12) $\frac{x}{\sqrt{1+2x}}$: write $\frac{x}{\sqrt{1+2x}} = \frac{\frac{1}{2}+x}{\sqrt{1+2x}} - \frac{\frac{1}{2}}{\sqrt{1+2x}} = \frac{1}{2}\sqrt{1+2x} - \frac{\frac{1}{2}}{\sqrt{1+2x}}$.
(13) $\frac{1}{(1+\sqrt{x})^4}$: use the substitution $x = u^2$. Then you'll have to integrate $\frac{2u}{(1+u)^4}$, which you can rewrite as $\frac{2u}{(1+u)^4} = \frac{2u+2}{(1+u)^4} - \frac{2}{(1+u)^4} = \frac{2}{(1+u)^3} - \frac{2}{(1+u)^4}$.

(The following two problems weren't on the worksheet.)

Problem 2. Compute the definite integral

$$\int_{-3}^{3} \left(x^5 - 6x^9 + \frac{\sin x}{(1+x^4)^2} \right) dx .$$

Hints. Let $f(x) = x^5 - 6x^9 + \frac{\sin x}{(1+x^4)^2}$. Then $f(x)$ is an odd function (i.e. $f(-x) = -f(x)$). Also, the lower bound of integration (-3) is the negative of the upper bound of integration (3). ☐

Problem 3. If $\int_0^6 f(x)\,dx = 10$ and $\int_0^4 f(x)\,dx = 7$, find $\int_4^6 f(x)\,dx$.

Hint. For any function $f(x)$ and numbers $a \leq b \leq c$, we have

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx\ .$$

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