# INDEFINITE INTEGRALS; FUNDAMENTAL THEOREM OF CALCULUS

WED, OCT 30, 2013

(*Last edited November 1, 2013 at 8:36am.*)

Summary of Fundamental Theorem of Calculus:

(1) Part 1 says that one (of many) antiderivatives of $f(x)$ is given by the function $F(x) = \int_a^x f(t)\,dt$. For this antiderivative, it is clear by the additive properties of the integral that $F(b) - F(a) = \int_a^b f(t)\,dt$.
(2) Part 2 says that if $G(x)$ is any other antiderivative of $f(x)$, then $G(b) - G(a) = \int_a^b f(t)\,dt$ also.

*Exercise* 1 (Section 5.3, #9). Use Part 1 of FTC to find the derivative of the function

$$g(s) = \int_5^s (t - t^2)^8 \; dt \ .$$

*Solution.* This is a straightforward application of FTC, which says that $g(x)$ is an antiderivative of the function $(x - x^2)^8$, or in other words $g'(x) = (x - x^2)^8$. □

*Exercise* 2 (Section 5.3, #13). Use Part 1 of FTC to find the derivative of the function

$$h(x) = \int_1^{e^x} \ln t \; dt \ .$$

*Solution.* Notice that $h(x)$ is the composite of two functions $f(x) = \int_1^x \ln t \; dt$ and $g(x) = e^x$, i.e. $h(x) = f(g(x))$. We know by FTC (part 1) that $f'(x) = \ln x$. So, by the Chain Rule, we have $f'(x) = h'(g(x))g'(x) = \ln(e^x)e^x = xe^x$.

(By the way, how can you use FTC (part 2) to solve this? Note that $F(x) = x \ln x - x$ is an antiderivative of $\ln x$. Thus $h(x) = F(e^x) - F(1) = (e^x \ln(e^x) - e^x) - (1 \ln 1 - 1) = xe^x - e^x + 1$. Thus $h'(x) = xe^x$.) □

*Exercise* 3 (Section 5.3, #31). Evaluate the integral

$$\int_0^{\pi/4} (\sec t)^2 \; dt \ .$$

*Solution.* Look up the fact that an antiderivative of $(\sec x)^2$ is $\tan x$. By FTC (part 2), we have

$$\int_0^{\pi/4} (\sec t)^2 \; dt = \tan \frac{\pi}{4} - \tan 0 = 1 - 0 = 1 \ .$$

□

*Exercise* 4 (Section 5.3, #40). Evaluate the integral

$$\int_1^2 \frac{4 + u^2}{u^3} \; du \ .$$

*Solution.* Notice that an antiderivative of $\frac{4+u^2}{u^3}$ is $\frac{-2}{u^2} + \ln u$. By FTC (part 2), we have

$$\int_1^2 \frac{4 + u^2}{u^3} \; du = \left( \frac{-2}{2^2} - \ln 2 \right) - \left( \frac{-2}{1^2} - \ln 1 \right) = \frac{3}{2} - \ln 2 \ .$$

□

*Exercise* 5 (Section 5.3, #72). If $f$ is continuous and $g$ and $h$ are differentiable functions, find a formula for

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\;dt\;.$$

*Solution.* Let $A(x) = \int_0^x f(t)\;dt$. By FTC (part 1), we have $A'(x) = f(x)$. Then

$$\int_{g(x)}^{h(x)} f(t)\;dt = \int_0^{h(x)} f(t)\;dt - \int_0^{g(x)} f(t)\;dt = A(h(x)) - A(g(x))$$

so

$$\begin{aligned}\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\;dt &= A'(h(x))h'(x) - A'(g(x))g'(x) \\ &= f(h(x))h'(x) - f(g(x))g'(x)\;.\end{aligned}$$

□