INDEFINITE INTEGRALS; FUNDAMENTAL THEOREM OF CALCULUS

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Let f be continuous on $[a,b]$. An *indefinite integral* (or *antiderivative*) of f is a function F which is continuous on $[a,b]$ and differentiable on (a,b) and $F'(x) = f(x)$ for all $x \in (a,b)$. By FTC (part 1), you can find an indefinite integral of f as follows: define $F(x) := \int_a^x f(t)dt$ for every real number x. Then

$$F(b) - F(a) = \int_a^b f(t)dt - \int_a^a f(t)dt = \int_a^b f(t)dt \ .$$

Part 2 of FTC says that $G(b) - G(a) = \int_a^b f(t)dt$ for any other indefinite integral G of f. This follows from the fact that any two indefinite integrals of f differ by a constant (if $F' = f$ and $G' = f$, then $(F-G)' = 0$, and any function whose derivative is 0 is a constant).

A definite integral is a number, while an indefinite integral is a function. Always take care to think about what type of integral is being dealt with. By the FTC, you can use indefinite integrals to compute definite integrals.

Exercise 1 (Section 5.3, #9). Use Part 1 of FTC to find the derivative of the function

$$g(s) = \int_5^s (t - t^2)^8 \ dt \ .$$

Exercise 2 (Section 5.3, #13). Use Part 1 of FTC to find the derivative of the function

$$h(x) = \int_1^{e^x} \ln t \ dt \ .$$

Exercise 3 (Section 5.3, #31). Evaluate the integral

$$\int_0^{\pi/4} (\sec t)^2 \ dt \ .$$

Exercise 4 (Section 5.3, #40). Evaluate the integral

$$\int_1^2 \frac{4 + u^2}{u^3} \ du \ .$$

Exercise 5 (Section 5.3, #72). If f is continuous and g and h are differentiable functions, find a formula for

$$\frac{d}{dx} \int_{g(x)}^{h(x)} f(t) \ dt \ .$$