DEFINITE INTEGRALS

MON, OCT 28, 2013

A *definite integral* is generally defined to be the limit of approximations of area. The formal definition is given on page 372, but it will usually suffice to use Theorem 4 on page 374, which says

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \quad \text{where} \quad \Delta x = \frac{b-a}{n} \quad \text{and} \quad x_i = a + i\Delta x \ . \tag{1}$$

In other words, the integral $\int_a^b f(x)dx$ is the limit of the sequence whose nth term is equal to the *Riemann sum* $\sum_{i=1}^{n} f(x_i)\Delta x$, which in turn is the sum of the areas of n rectangles, where the ith rectangle has width $\Delta x = \frac{b-a}{n}$ and (possibly negative) height $f(x_i) = f(a + i\Delta x)$.

Exercise 1 (Section 5.2, #17). Express the limit $\lim_{n\to\infty} \sum_{i=1}^{n} x_i \ln(1 + x_i^2)\Delta x$, where $\Delta x = \frac{6-2}{n}$ and $x_i = 2 + i\Delta x$, as a definite integral on the interval $[2, 6]$.

Exercise 2 (Section 5.2, #24). Use (1) to compute $\int_0^2 (2x - x^3)dx$.

Exercise 3. Use (1) to compute $\int_0^4 e^x dx$.

Exercise 4 (Section 5.2, #27). Prove that $\int_a^b x \ dx = \frac{1}{2}(b^2 - a^2)$. Note that this is $f(b) - f(a)$ where $f(x) = \frac{1}{2}x^2$.

Exercise 5 (Section 5.2, #28). Prove that $\int_a^b x^2 \ dx = \frac{1}{3}(b^3 - a^3)$. Note that this is $f(b) - f(a)$ where $f(x) = \frac{1}{3}x^3$.

Exercise 6 (Section 5.2, #29). Express the integral $\int_1^{10} (x - 4\ln x) \ dx$ as a limit of Riemann sums as in (1).

Exercise 7 (Section 5.2, #57). Use the properties of integrals (page 379 to 381) to verify the following inequality without evaluating an integral:

$$2 \le \int_{-1}^{1} \sqrt{1 + x^2} \ dx \le 2\sqrt{2} \ .$$