MORE FUNCTION GRAPHING; OPTIMIZATION

FRI, OCT 25, 2013

Exercise 1. Let n be an arbitrary positive integer. Give an example of a function with exactly n vertical asymptotes. Give an example of a function with infinitely many vertical asymptotes.

Exercise 2. Let f be a function which is differentiable everywhere. Suppose that $f'(x) > 1$ for all x. Show that $\lim_{x \to \infty} f(x) = \infty$.

Exercise 3. Graph the function

$$f(x) = x^3 + 6x^2 + 9x \ .$$

Indicate domain, critical points, inflection points, regions where the graph is increasing/decreasing, x-intercepts and y-intercepts, regions of concavity (up or down), local maxima and minima, any asymptotes and behavior at infinity.

Exercise 4. Find

$$\lim_{t \to 16} \frac{\sqrt{t} - 4}{t - 16}$$

in three ways: (i) using methods learned up to and including the first midterm; (ii) by realizing the limit as $f'(c)$ for some function $f(t)$ and some value c; (iii) using L'Hospital's Rule.

Exercise 5 (Section 4.7, #19). Find the point on the line $y = 2x + 3$ that is closest to the origin.

Exercise 6 (Section 4.7, #21). Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$.

Exercise 7 (Section 4.7, #24). Find the area of the largest rectangle that can be inscribed in the ellipse $x^2/a^2 + y^2/b^2 = 1$.

Exercise 8 (Section 4.7, #54). At which points on the curve $y = 1 + 40x^3 - 3x^5$ does the tangent line have the largest slope?