MATH 1A WORKSHEET

(1) Find the derivative of the following functions.
    (a) $f(x) = x^{\cos x}$, defined on the domain $(0, \infty)$
    (b) $f(x) = \log_5(3x^2 - 2)$, defined on the domain $(-\infty, -\sqrt{2/3}) \cup (\sqrt{2/3}, \infty)$
    (c) $f(x) = (\sqrt{x})^x$, defined on the domain $(0, \infty)$

(2) Let f be the function $f(x) = \ln(x - 1)$, defined on the domain $(1, \infty)$. Find $\frac{d^n f}{dx^n}$ for any positive integer n.

(3) Find $\frac{dy}{dx}$ by implicit differentiation.
    (a) $x^3 + y^3 = 6xy$
    (b) $x \sin y + y \sin x = 1$

(4) Find all points on the curve $x^2 y^2 + xy = 2$ where the slope of the tangent line is -1.

(5) Draw a graph of the curve $x^2 - y^2 = 1$. Use implicit differentiation to find the tangent line at all points except $(-1, 0)$ and $(1, 0)$. What goes wrong for these points? What is the tangent line to the curve at the point $(1, 0)$?

(6) Draw a graph of the curve $x^2 - y^2 = 0$. Argue that there isn't a good way to define the "tangent line" to the curve at the point $(0, 0)$.

(7) Draw a graph of the curve $y^2 - x^4 = 0$. Use implicit differentiation to find the tangent line at all points except $(0, 0)$. What goes wrong for $(0, 0)$? Find the tangent line to the curve at $(0, 0)$.