PRACTICE MIDTERM 2

Write in complete sentences and show all work.

Problem 1. (i) Complete the sentence: "A function f is continuous at $x = a$ if ..." Give an example of a function f and some a in the domain of f such that f is not continuous at $x = a$.

(ii) Complete the sentence: "A function f is differentiable at $x = a$ if ..." Give an example of a function f and some a in the domain of f such that f is not differentiable at $x = a$.

Problem 2. (i) Find the equation of the tangent line to the curve $y = \frac{2\ln x}{x^3} - (\sin^{-1}(x))^5$ at the point where $x = \frac{1}{2}$.

(ii) Find the tangent line to the graph $x^2 + 2y^4 = 3$ at the point $(1, 1)$.

Problem 3. (i) Let $f(x)$ be a function of x such that $(f(x))^x = 1$. Find $f'(x)$ in terms of f.

(ii) Let f, g be differentiable functions such that $g(x) > 0$ for all x. Set $h(x) = f(x^{g(x)})$. Find h' in terms of f, g, f', g'.

(iii) Let $f(x) = 3x$. Find a function F whose derivative is f.

Problem 4. Strontium-90 has a half-life of 28 days. A sample has a mass of 50 mg initially.

(a) How much of the sample remains after t days?

(b) How long does it take the sample to decay to a mass of 2 mg?

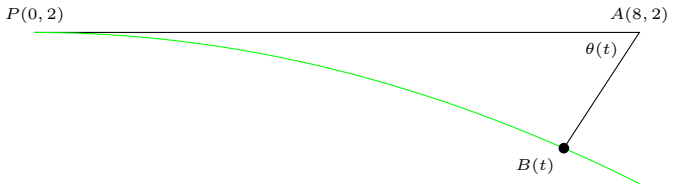
Problem 5. (i) State Rolle's Theorem.

(ii) Use Rolle's Theorem to prove the Mean Value Theorem.

(iii) Let $f(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x$. Prove that $f(\pi) > f(1)$.

Problem 6. Find $\lim_{x\to 0^+} x \ln x$.

Problem 7. A pitcher at $P = (0, 2)$ throws a ball whose position at time t is $B(t) = (x(t), y(t))$ where $x(t) = t$ and $y(t) = -\frac{1}{32}(x(t))^2 + 2$. You are watching the ball from the point $A = (8, 2)$. Let $\theta(t)$ be the angle $\angle PAB(t)$. How fast (in radians) is $\theta(t)$ changing when the ball hits home plate, i.e. when $B(t) = (8, 0)$?



Problem 8. Consider the graph of the function $f(x) = x^3 - x$.

(a) Find the regions of the graph where $f(x)$ is increasing. Find the regions of the graph where $f(x)$ is decreasing. Find all local extrema of f. Which of these are global extrema?

(b) Find the regions of the graph where $f(x)$ is concave up. Find the regions of the graph where $f(x)$ is concave down. Find all inflection points.