

## LIMIT DEFINITIONS

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The statement

$$\text{“} \lim_{x \rightarrow (Q)} f(x) = (P) \text{”} \tag{1}$$

is defined as

$$\text{“For every (A), there exists (B) such that if (C) then (D)”} \tag{2}$$

using the following tables:

(P)	(A)	(D)	(Q)	(B)	(C)
$L$	$\epsilon > 0$	$ f(x) - L  < \epsilon$	$a$	$\delta > 0$	$0 <  x - a  < \delta$
$\infty$	$N > 0$	$f(x) > N$	$a^+$	$\delta > 0$	$0 < x - a < \delta$
$-\infty$	$N < 0$	$f(x) < N$	$a^-$	$\delta > 0$	$-\delta < x - a < 0$
			$\infty$	$M > 0$	$x > M$
			$-\infty$	$M < 0$	$x < M$

For example, the statement

$$\text{“} \lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty \text{”}$$

is defined as

$$\text{“For every } N < 0, \text{ there exists } \delta > 0 \text{ such that if } -\delta < x - 0 < 0 \text{ then } \frac{1}{x} < N \text{”}$$