A Lasserre-based $(1+\varepsilon)$-approximation for Makespan Scheduling with Precedence Constraints

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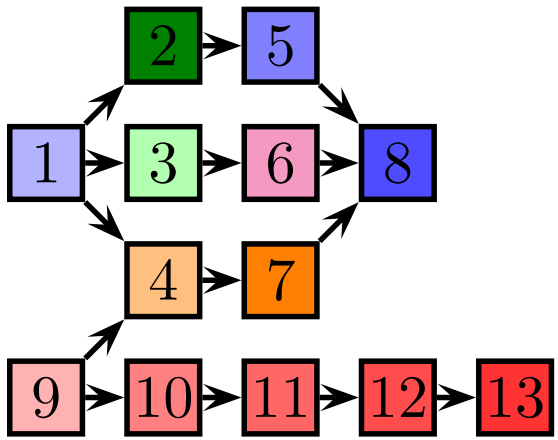
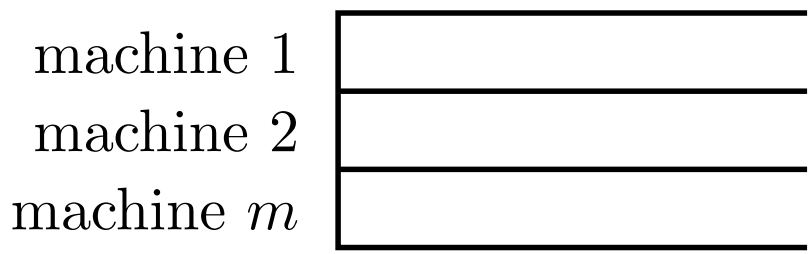
UNIVERSITY of WASHINGTON

Makespan Scheduling w. Precedence Constraints

Input:

- Unit-size jobs J with precedence constraints
- Number m of identical machines





Makespan Scheduling w. Precedence Constraints

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- Unit-size jobs J with precedence constraints
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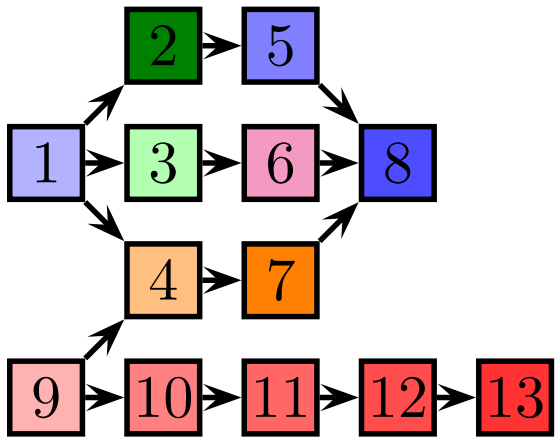
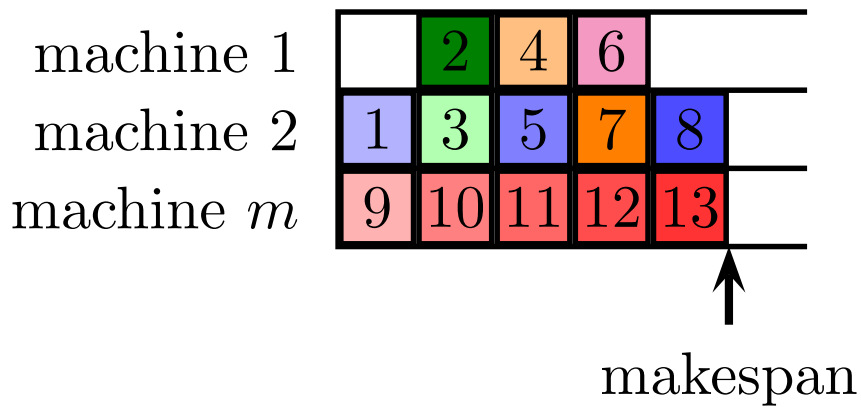
Goal: Schedule jobs as to **minimize makespan**





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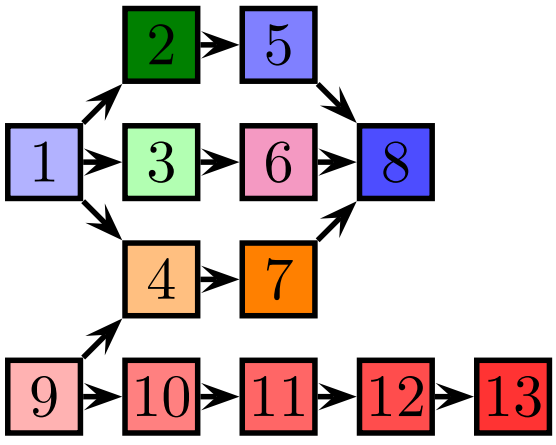

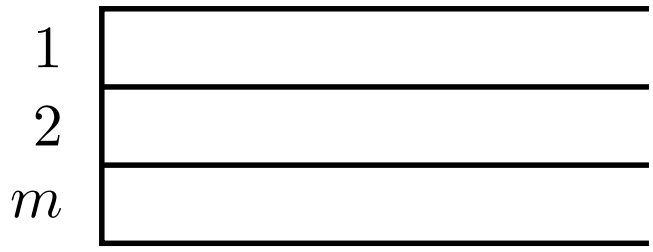

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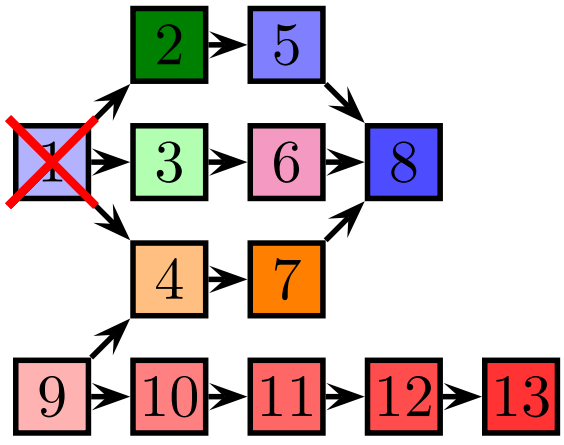

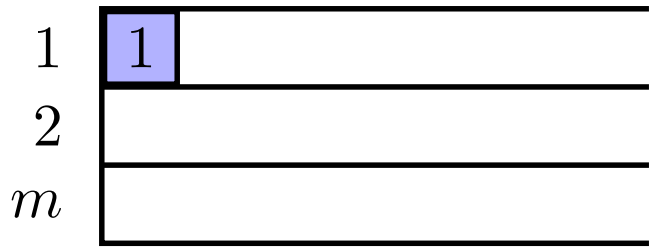

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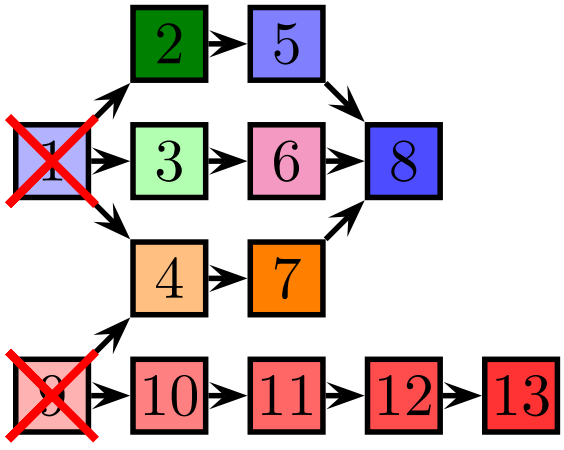

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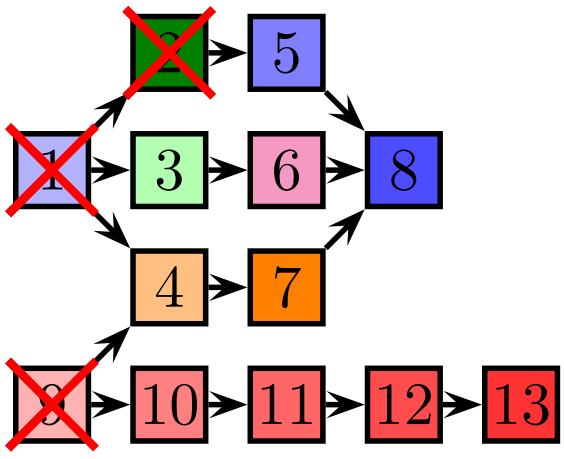

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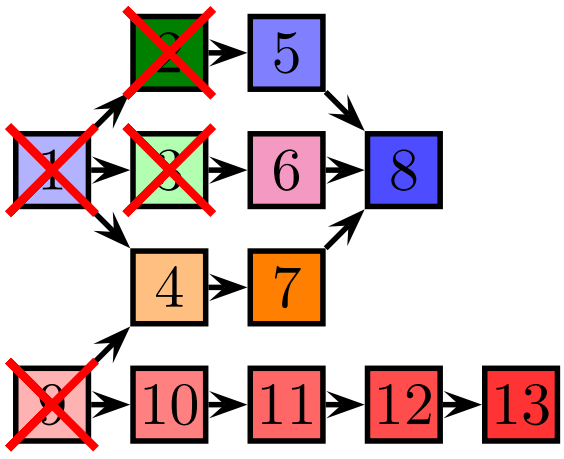

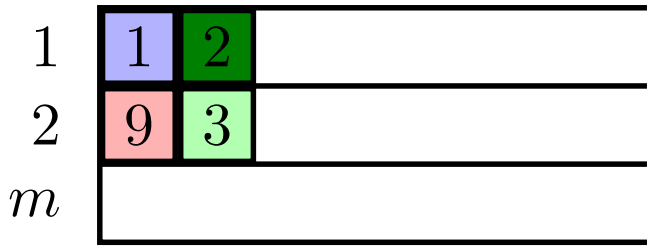

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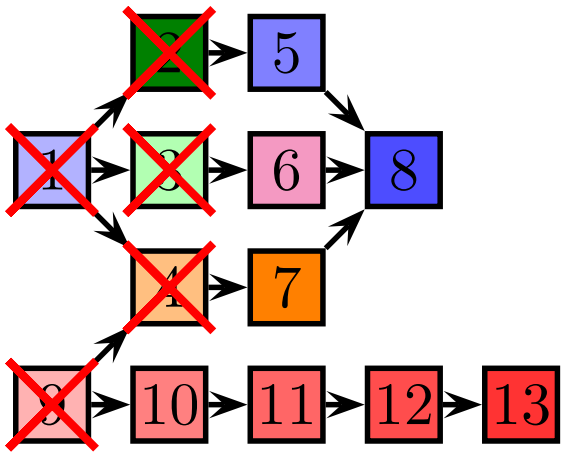

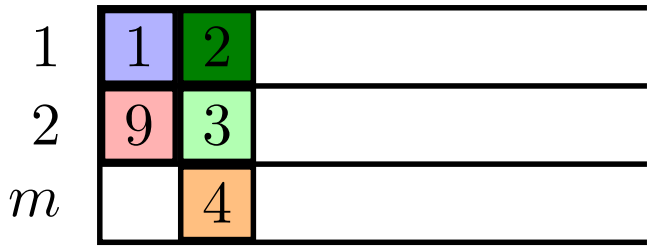

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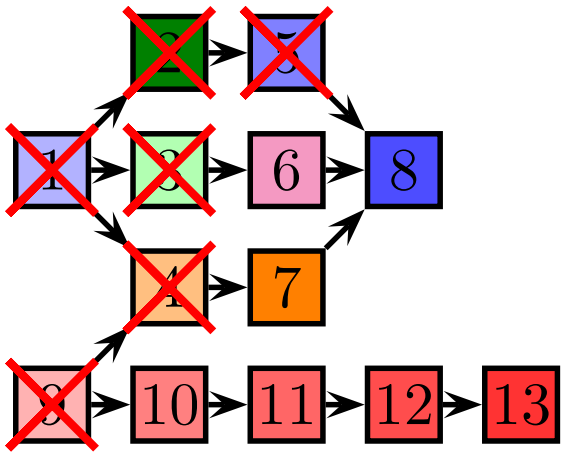

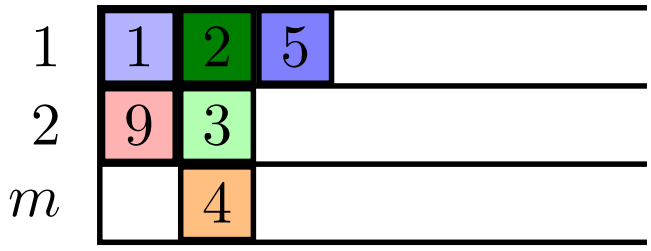

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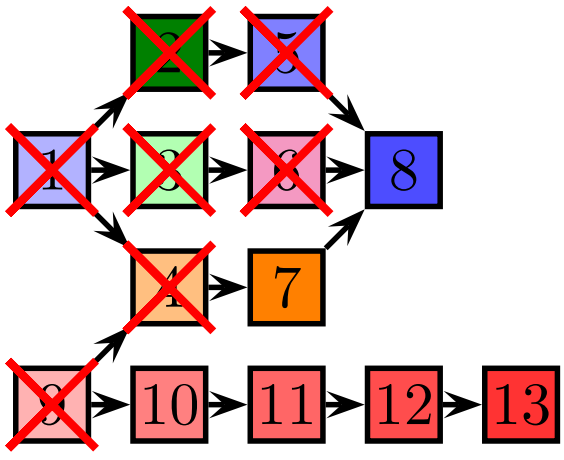

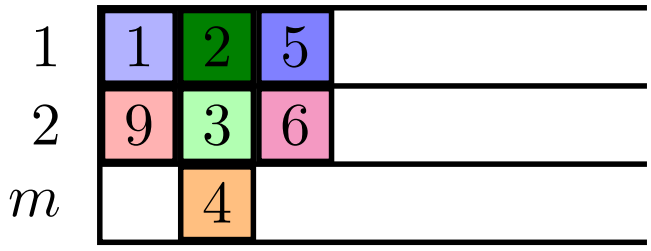

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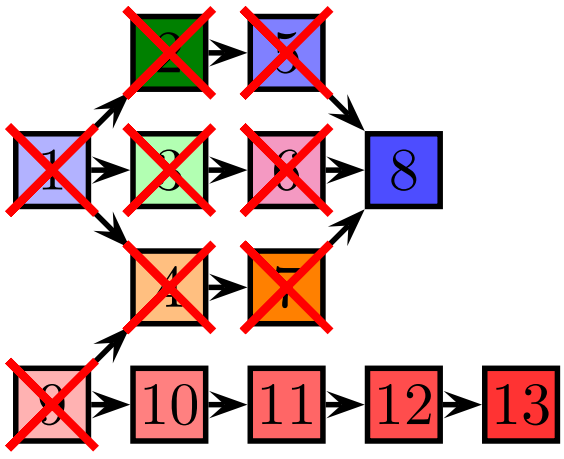

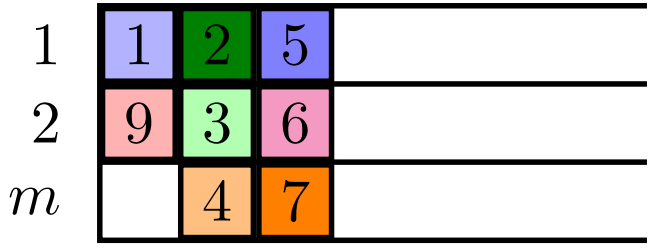

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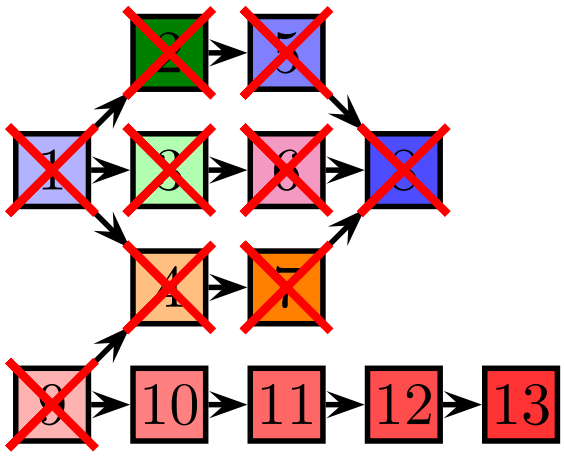

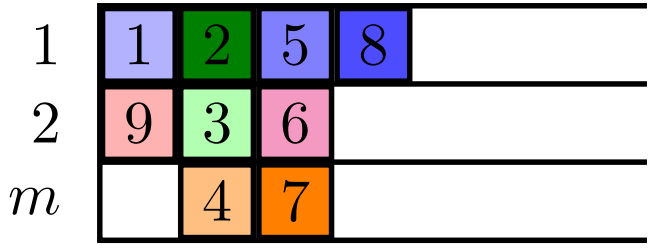

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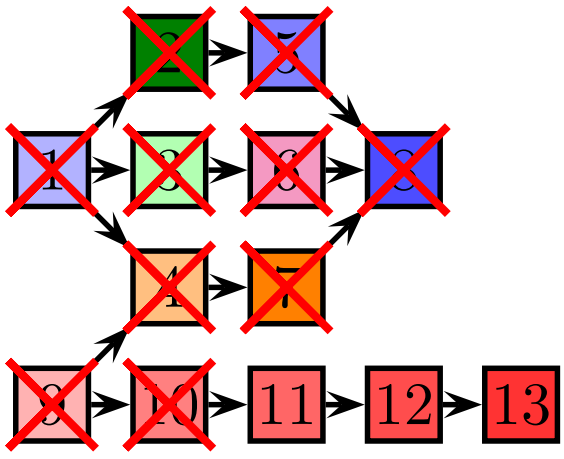

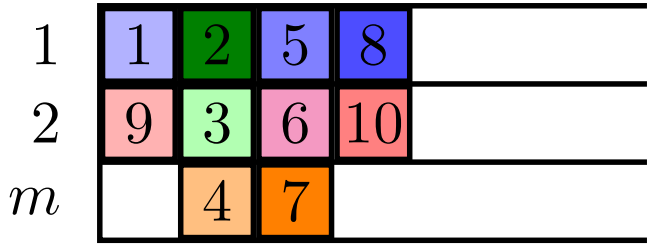

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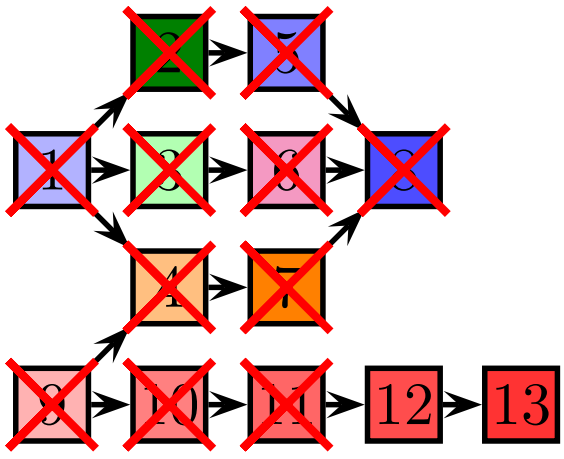

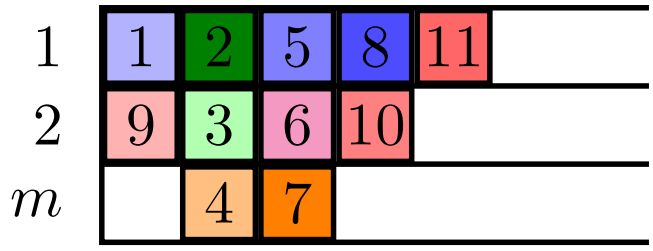

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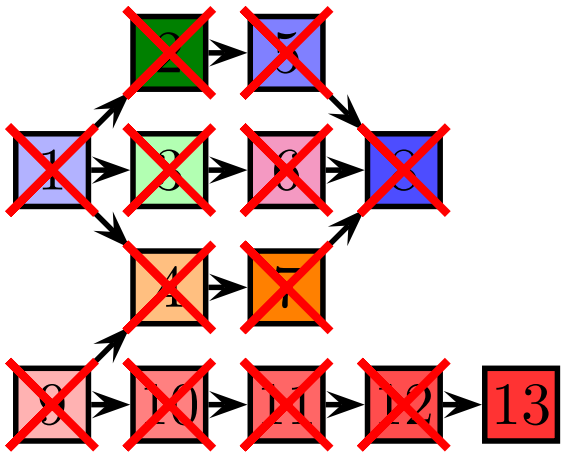

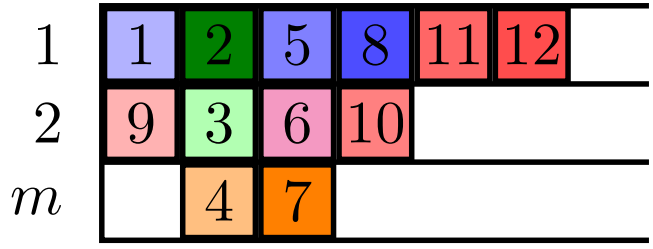

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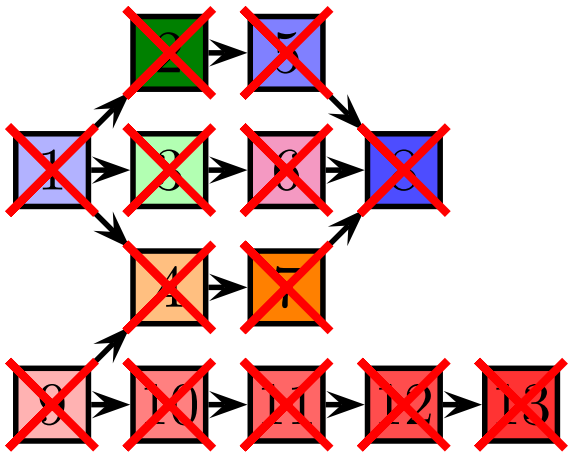

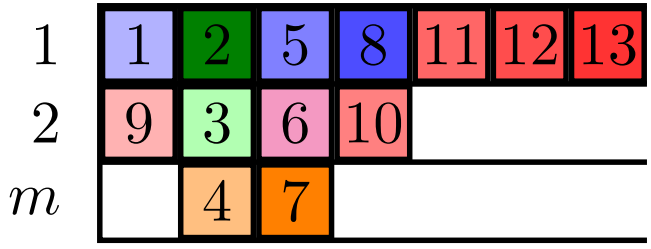

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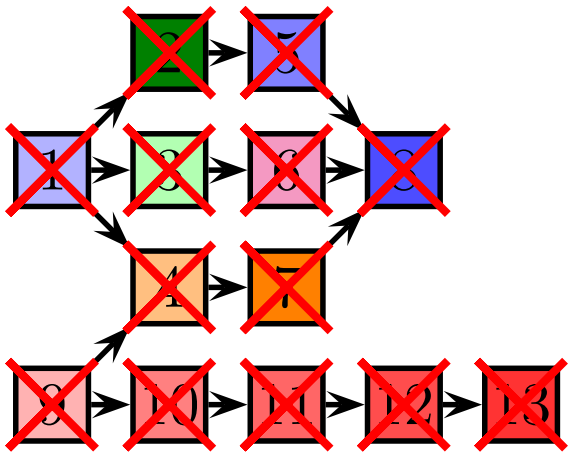

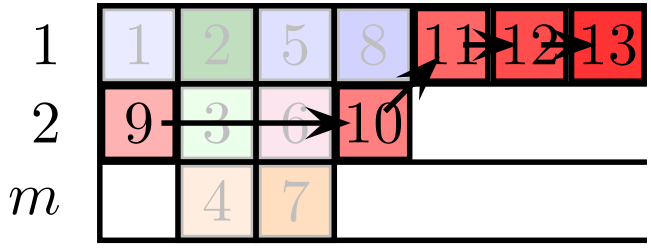

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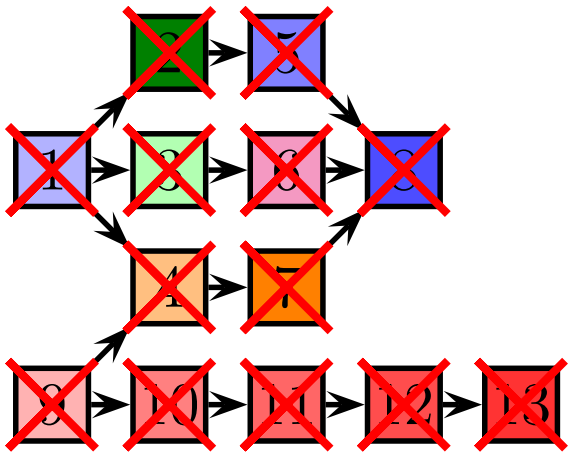

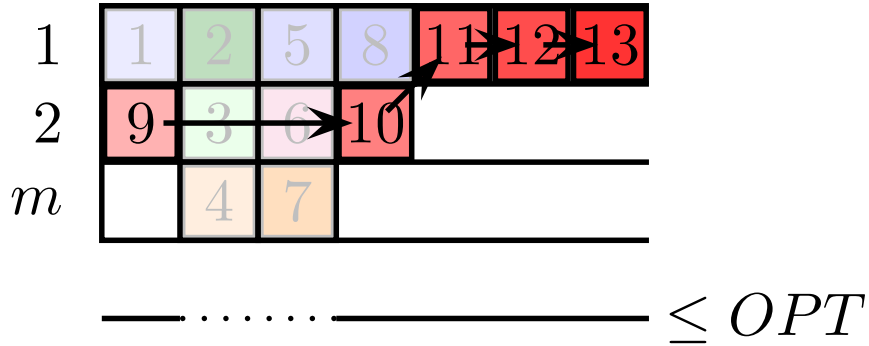

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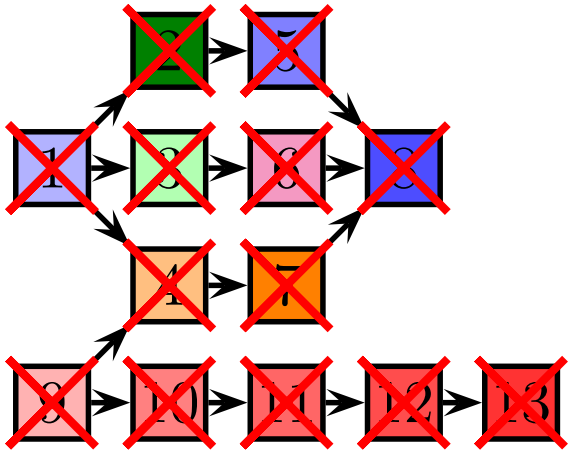

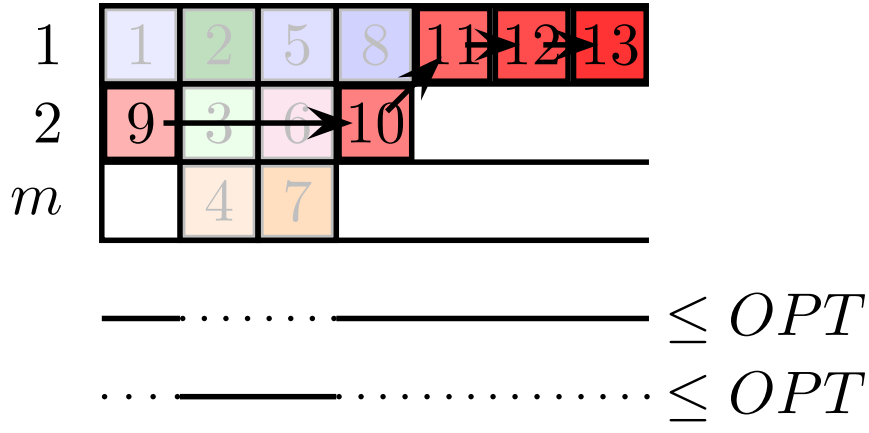

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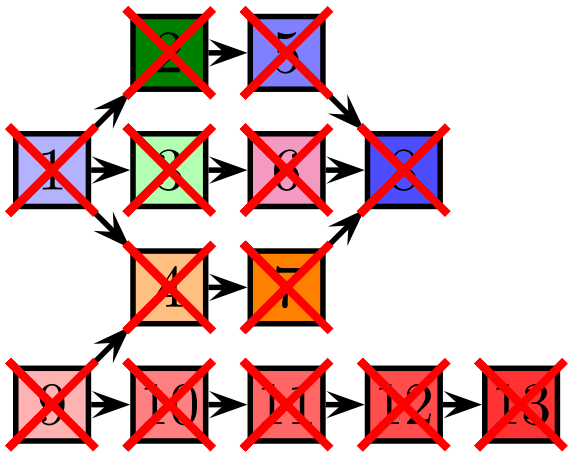

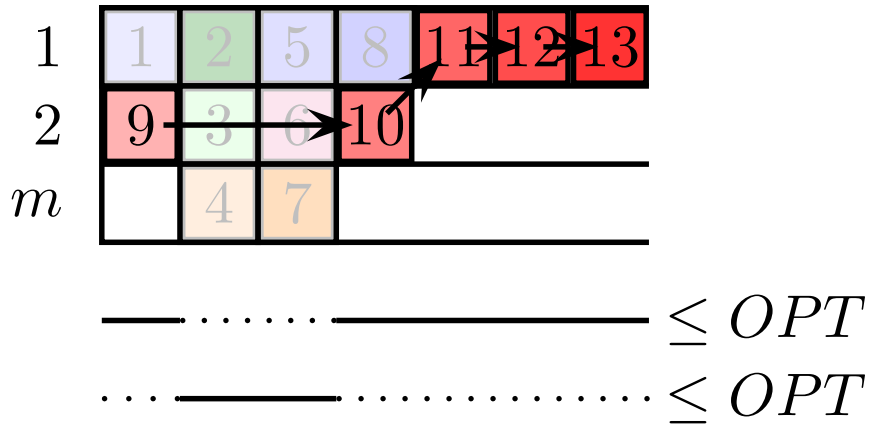

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$\Rightarrow$ 2-apx ($(2 - \frac{2}{m})$ for $p_j = 1$; $2 - \frac{1}{m}$ for arbitrary p_j)

A linear programming formulation

$$
\begin{aligned}
\sum_{t=1}^{T} x_{jt} &= 1 \quad \forall j \in J \\
\sum_{j \in J} x_{jt} &\leq m \quad \forall t \in [T] \\
\sum_{t' \leq t} x_{it'} &\leq \sum_{t' \leq t+1} x_{jt'} \quad \forall i \prec j \; \forall t \in [T] \\
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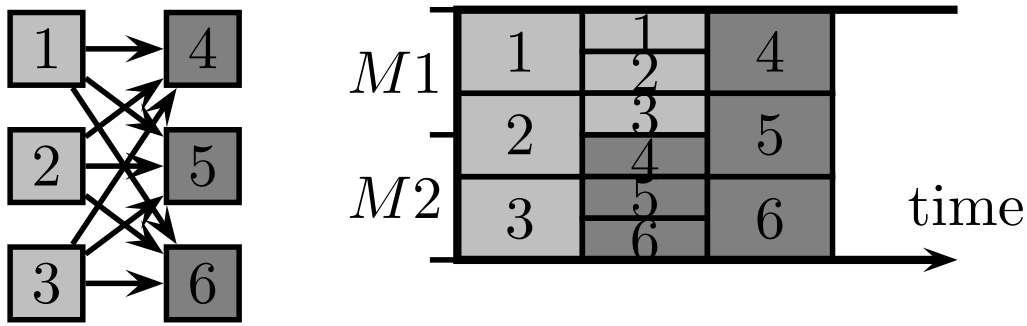

- Integrality gap: $2 - \Theta(\frac{1}{m})$

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Unit size job makespan minimization:

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- Is $P3 \mid p_j = 1, \text{prec} \mid C_{\max}$ **NP**-hard? [Garey Johnson '79]

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  [Part of # 1 of "Open problems in Scheduling";
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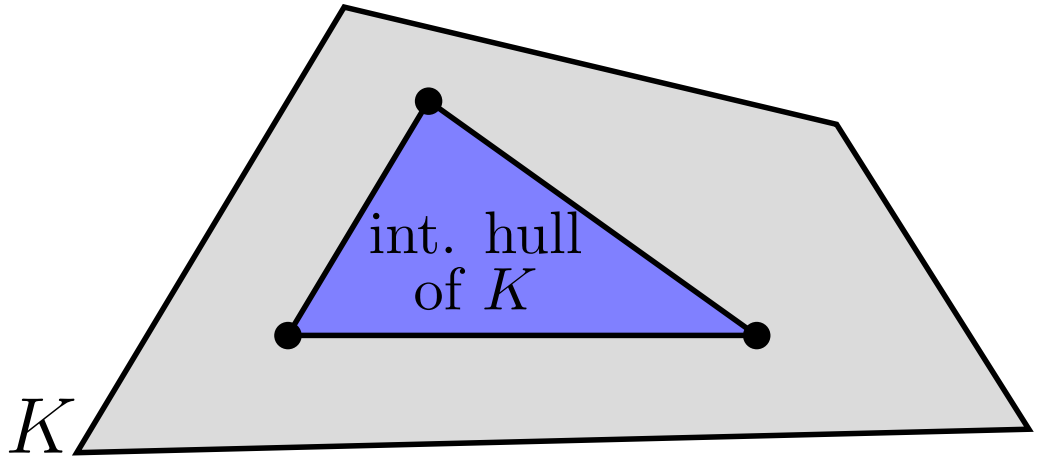
Theorem (Levey, R. 15)

The integrality gap of LP is $1 + \varepsilon$ after $(\log n)^{O(\log \log n)}$**-Lasserre rounds** (for constants m and $\varepsilon > 0$).

- Running time $n^{(\log n)^{O(\log \log n)}}$
- Sherali-Adams suffices

Lift-and-project methods



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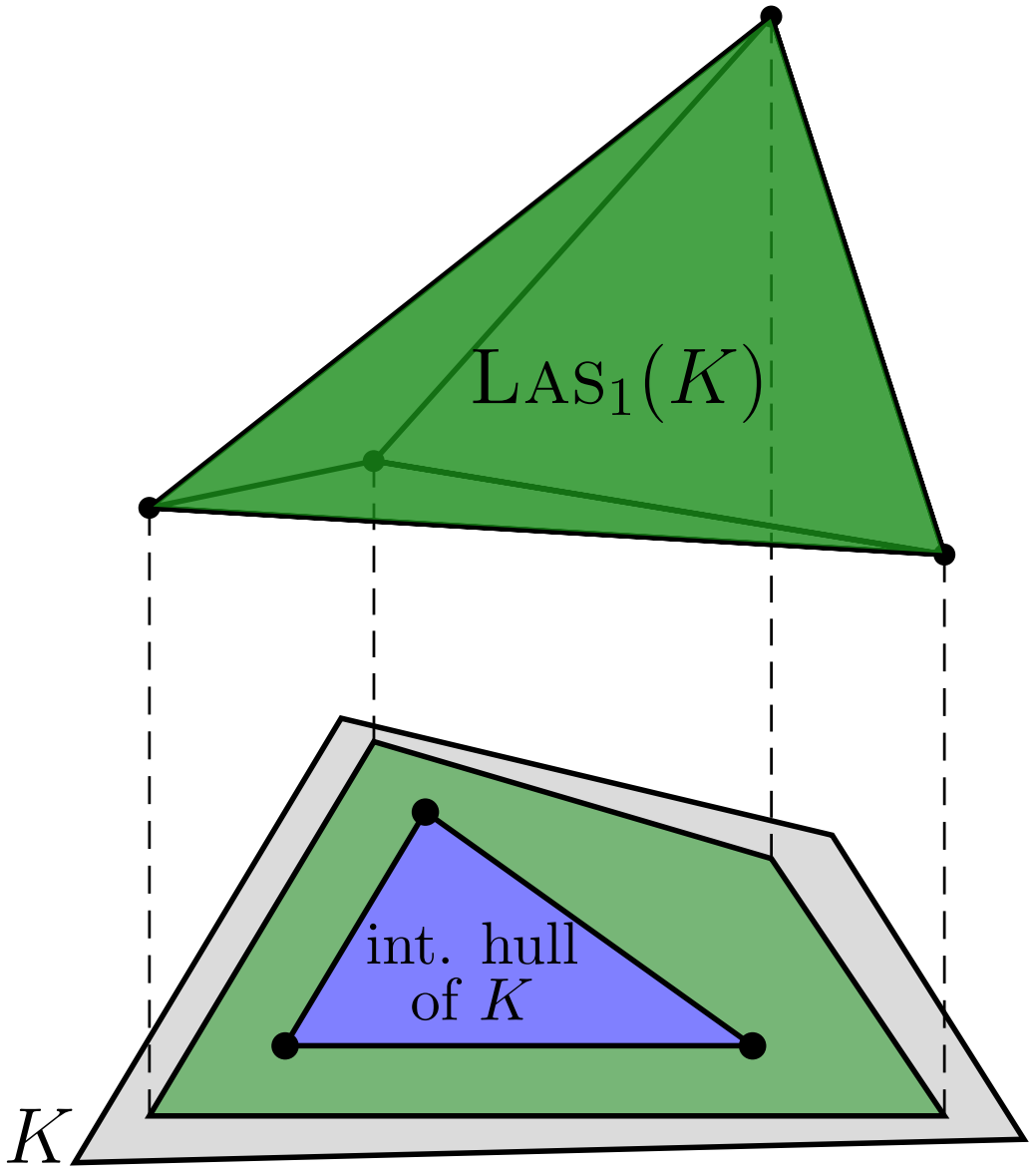

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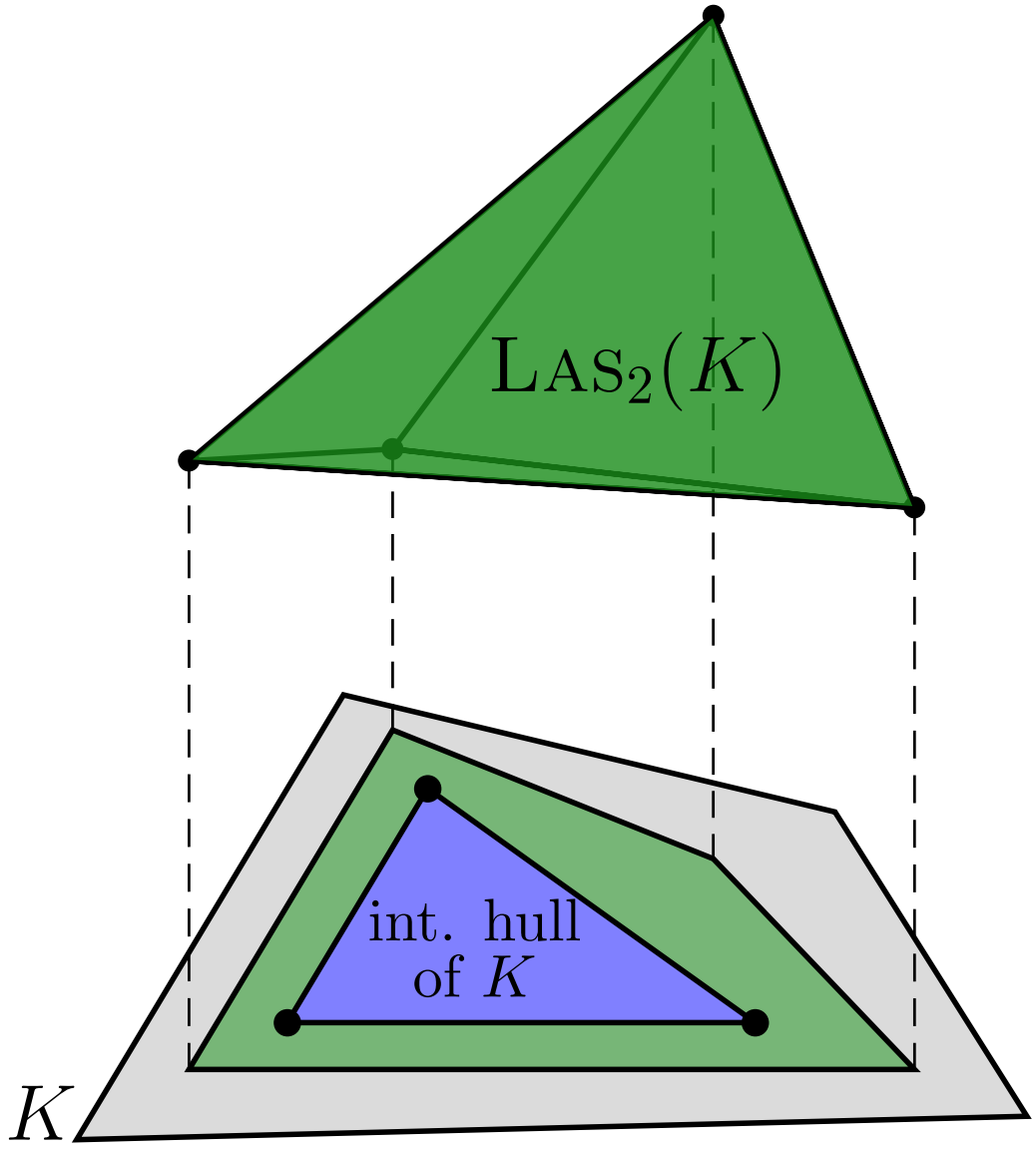

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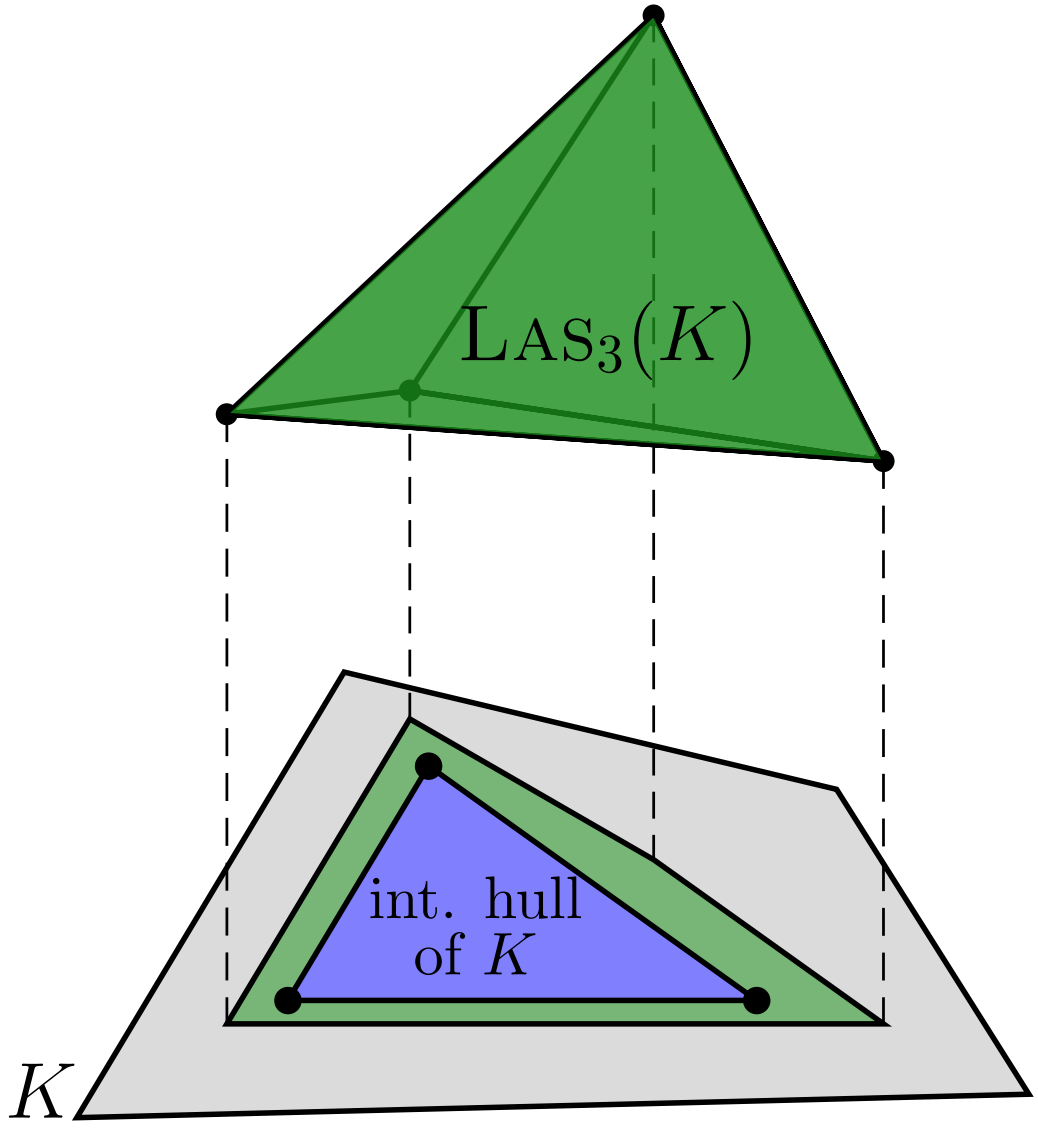

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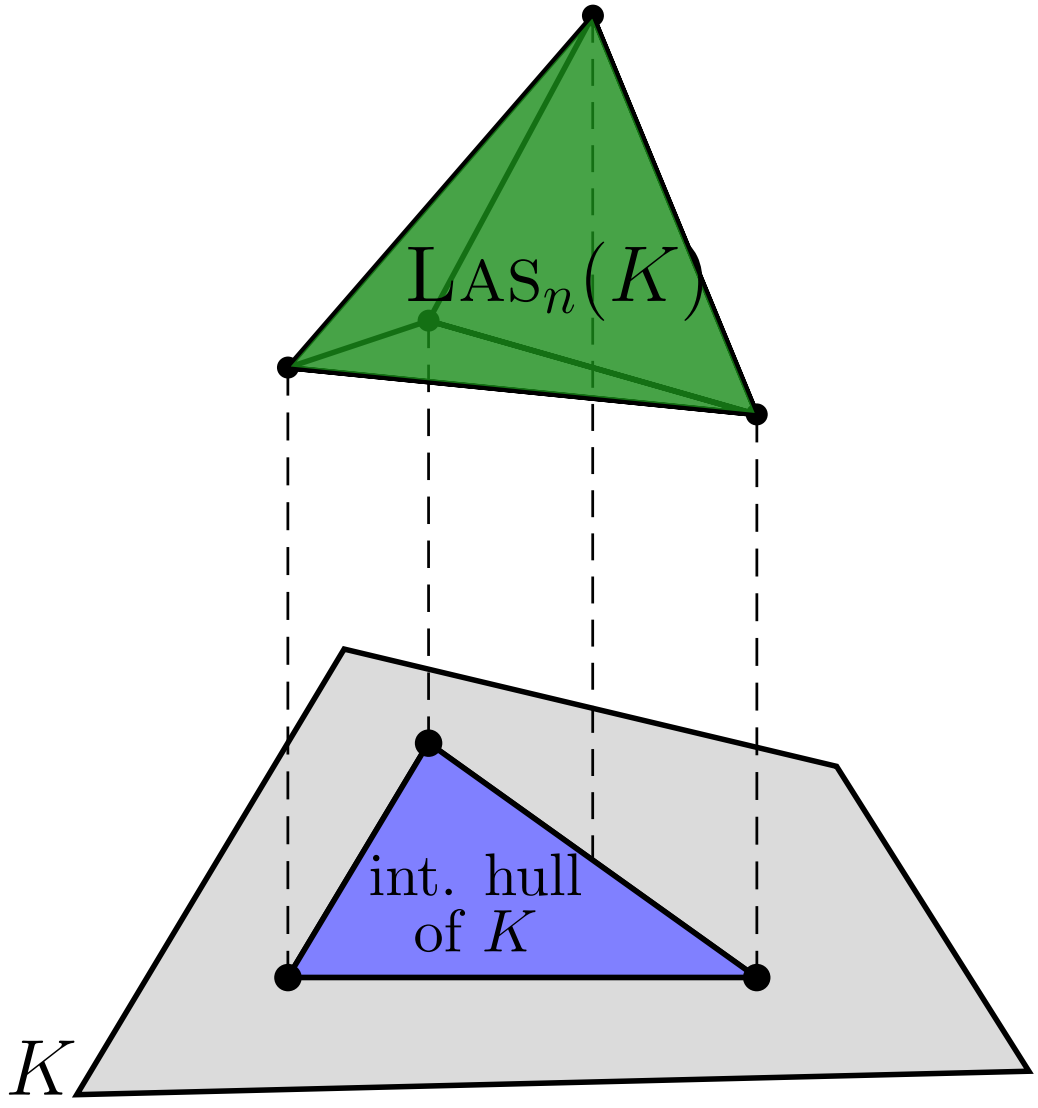

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$$y_I \equiv \bigwedge_{i \in I} (x_i = 1)$$

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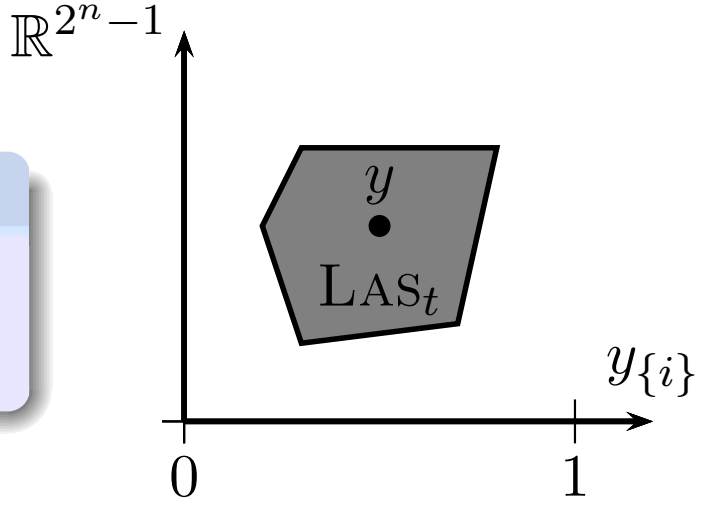
- Solvable in time $n^{O(t)} m^{O(1)}$

Inducing on one variable

Lemma

For $y \in \text{LAS}_t(K)$, $t \geq 1$, $i \in [n]$,
$y \in conv\{z \in \text{LAS}_{t-1}(K) \mid z_i \in \{0,1\}\}$.



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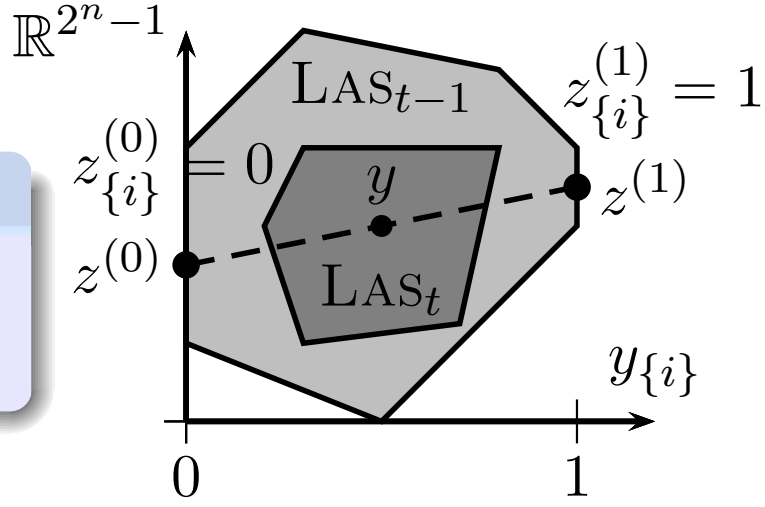

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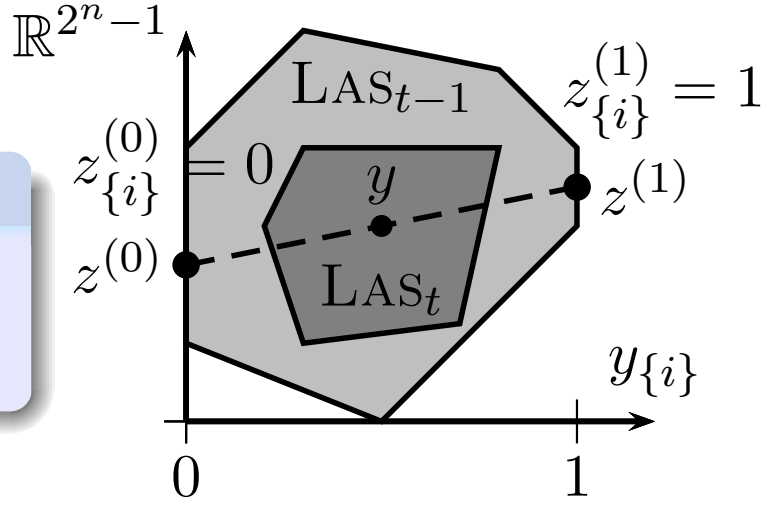
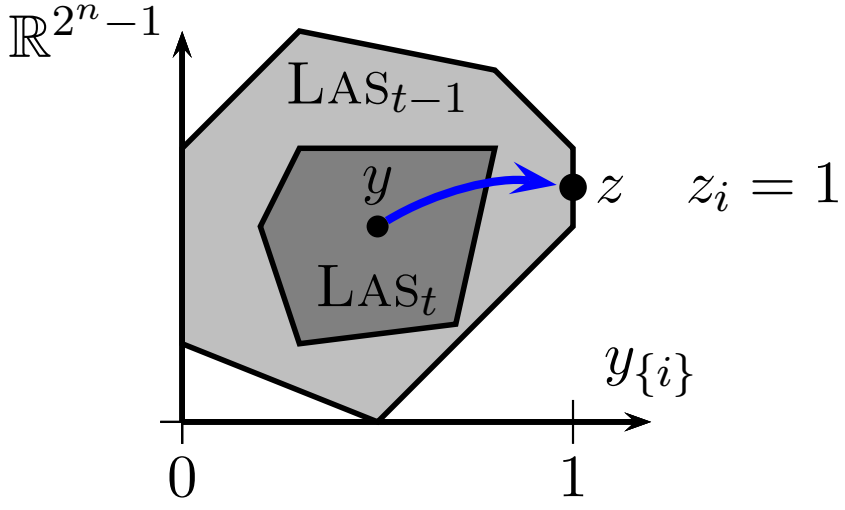


Operation: "Induce on $x_i = 1$"



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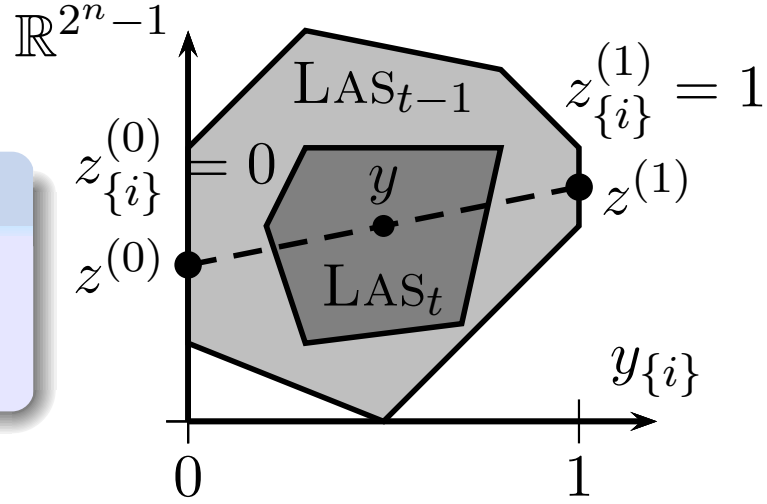

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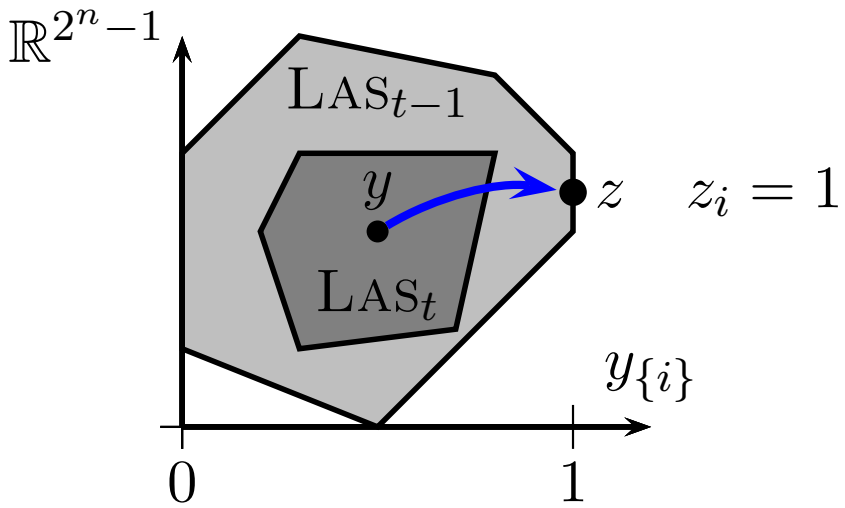
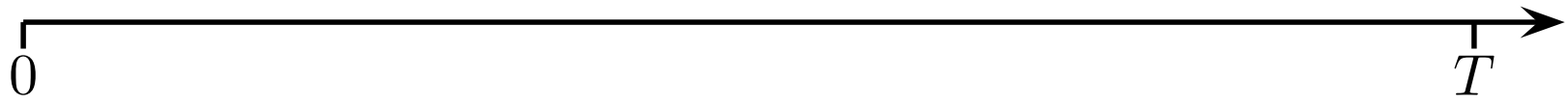


- **Observation:** Conditioning only shrinks support!

Partition



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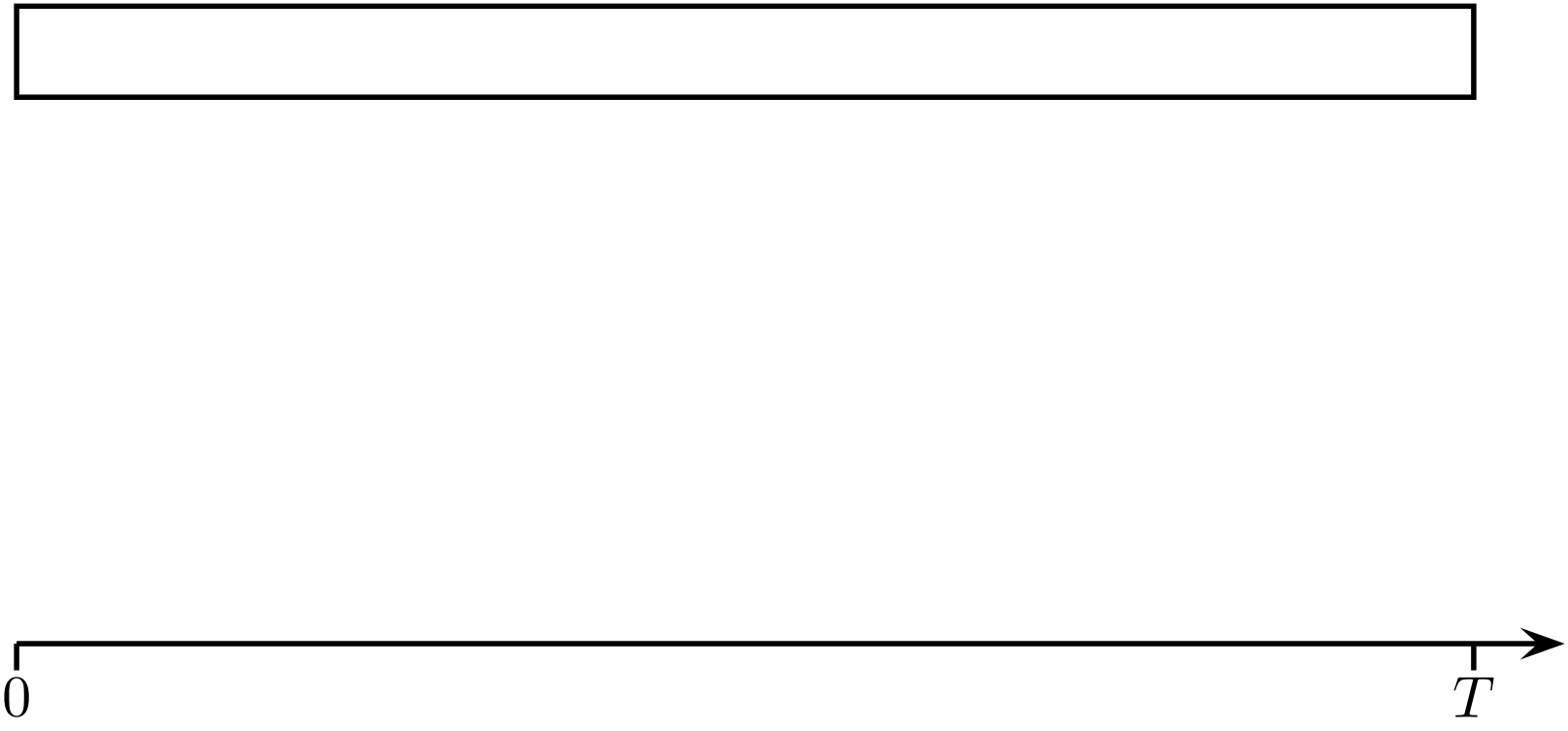
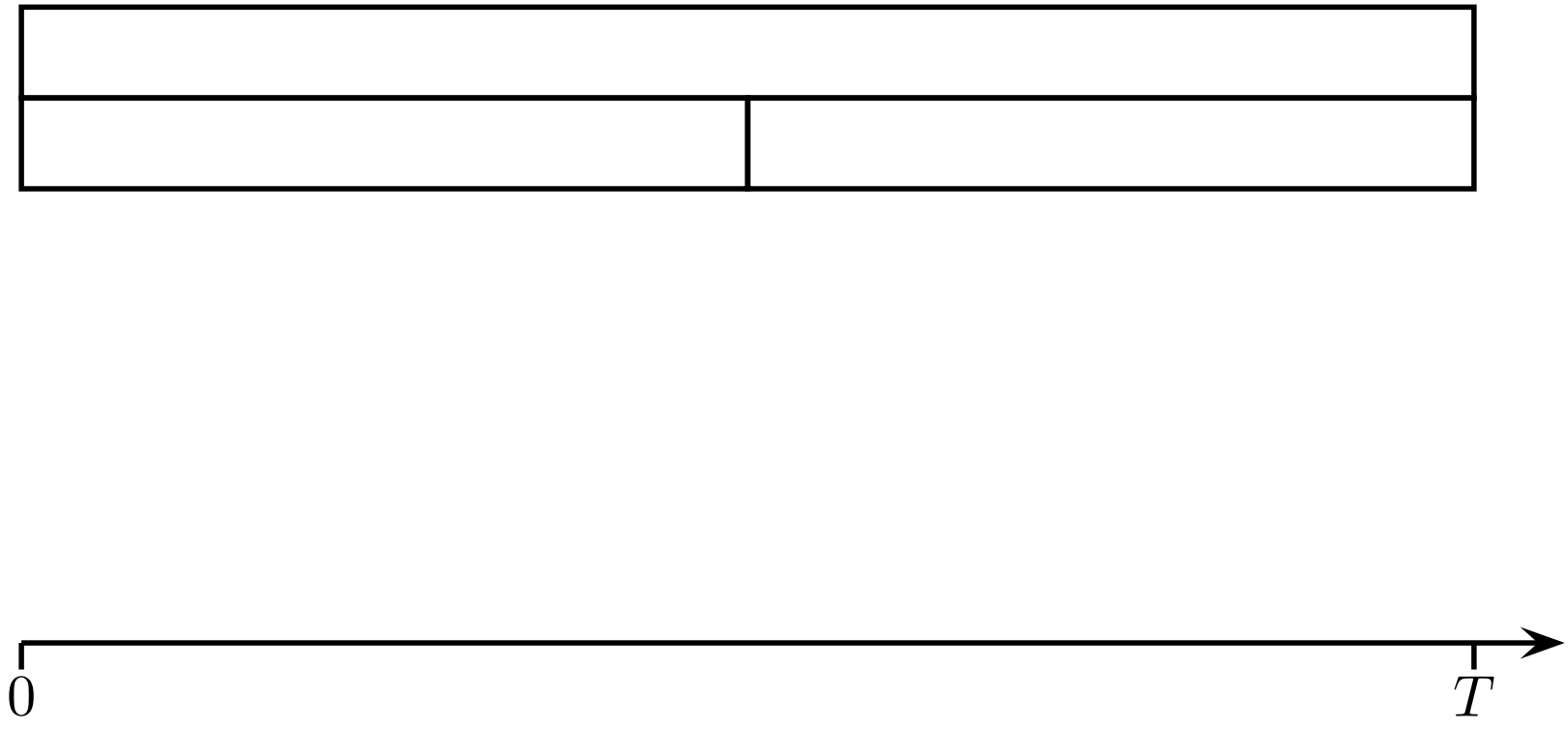


- Partition time horizon into binary family

Partition



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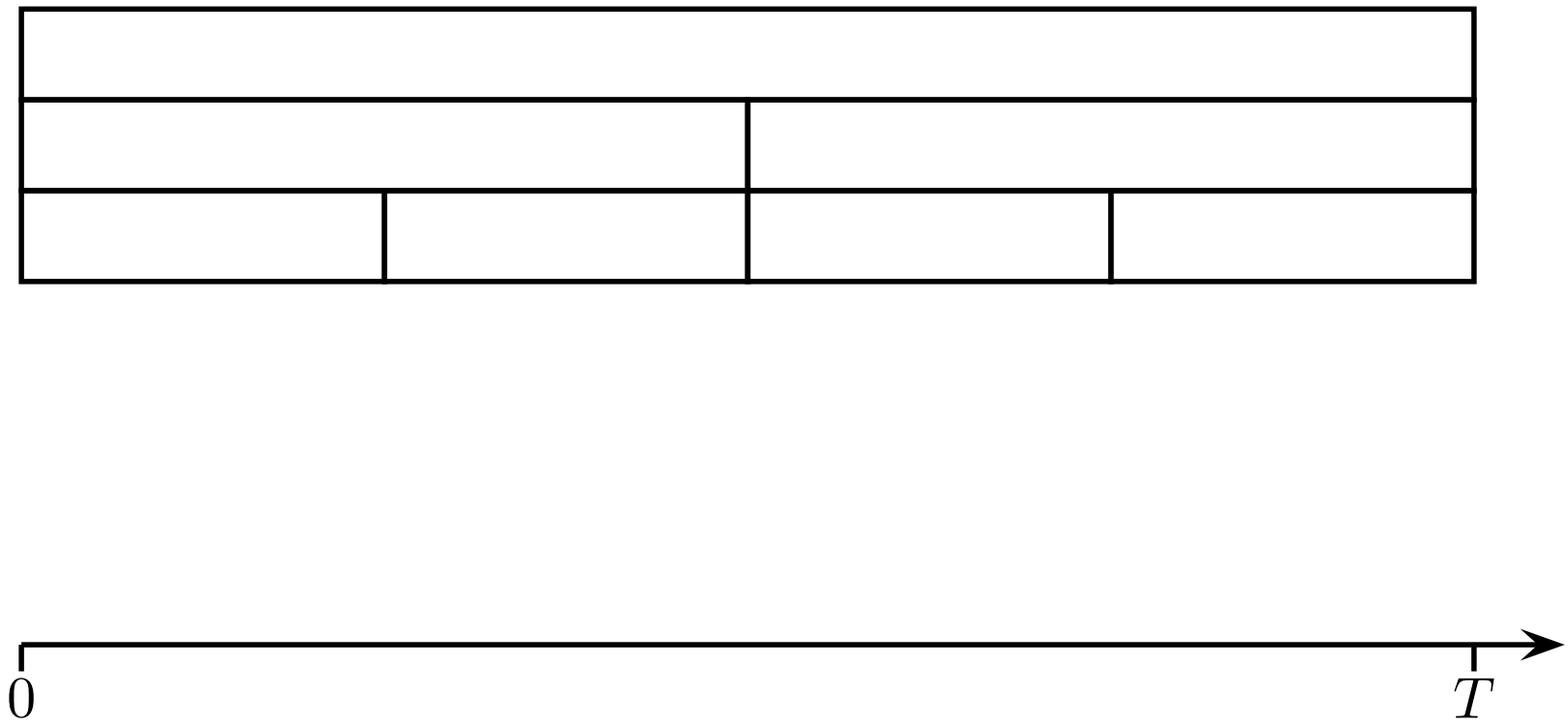

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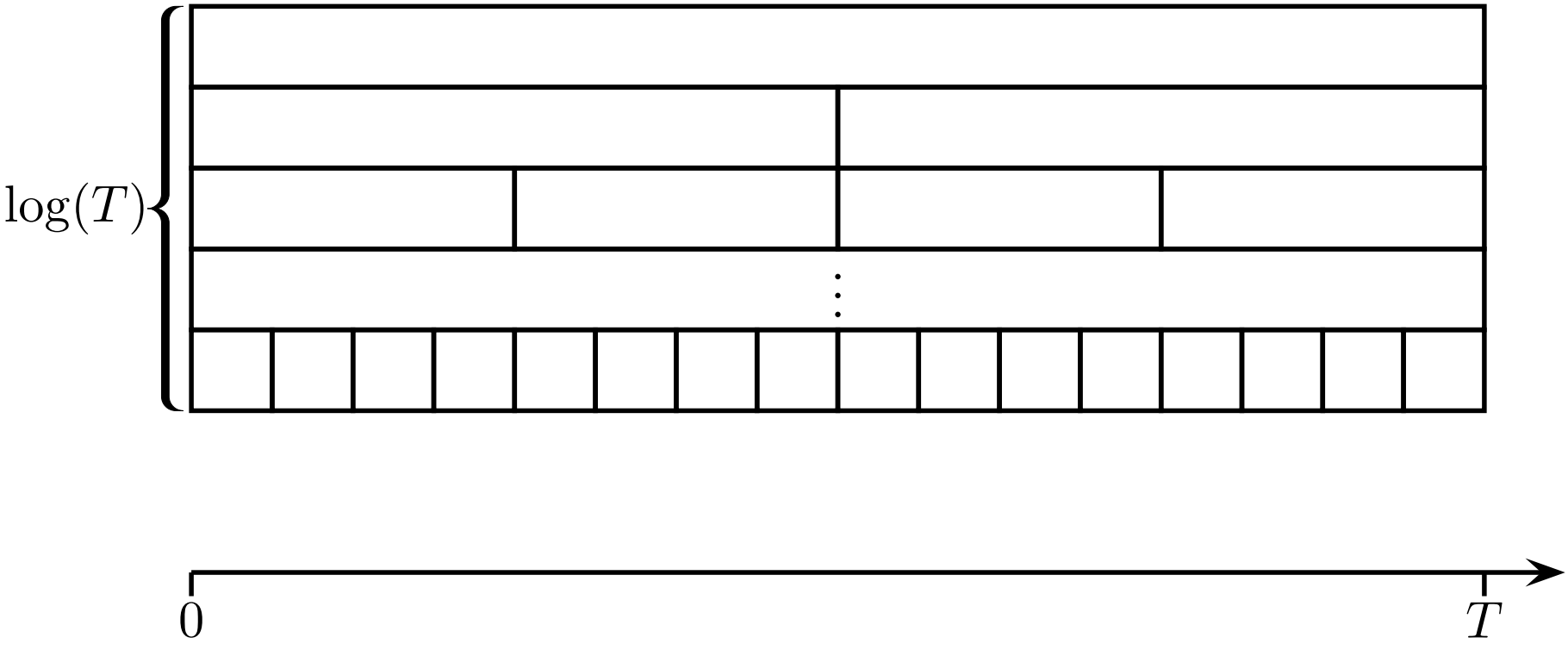

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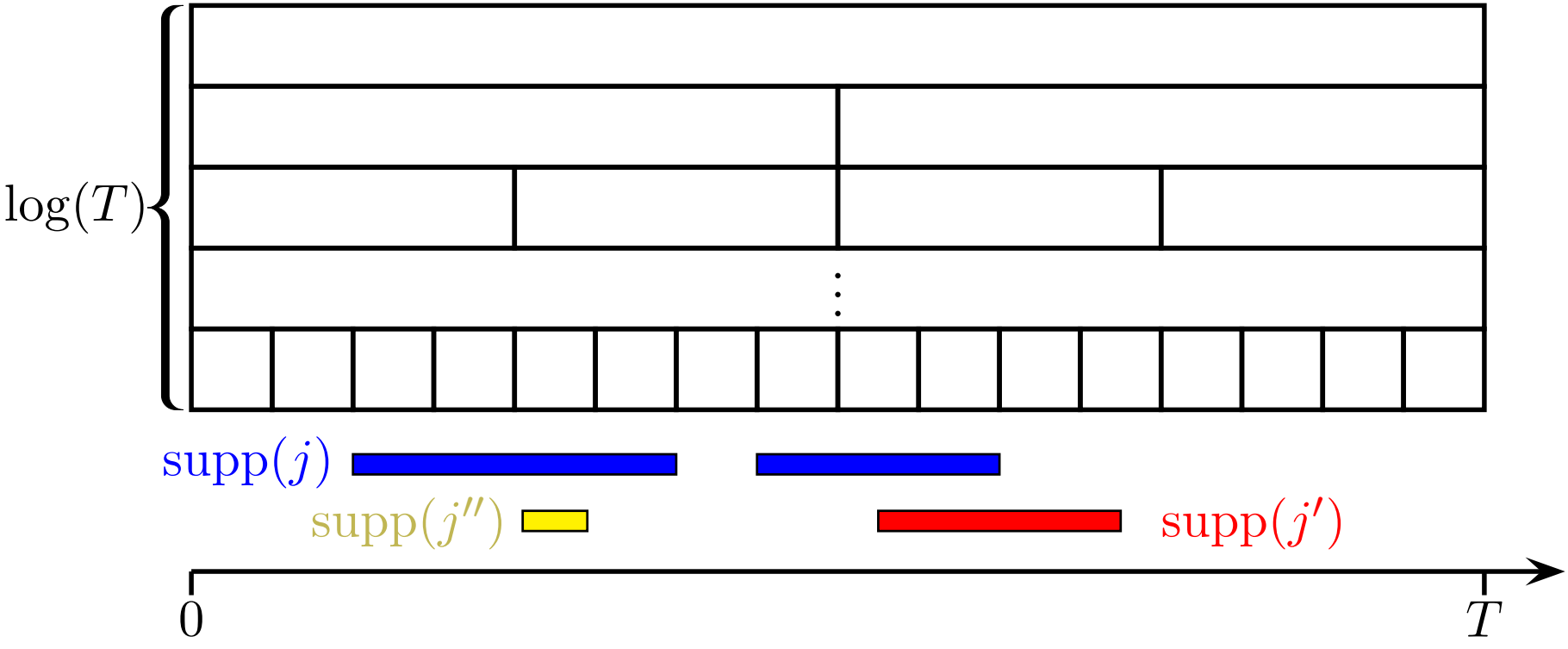

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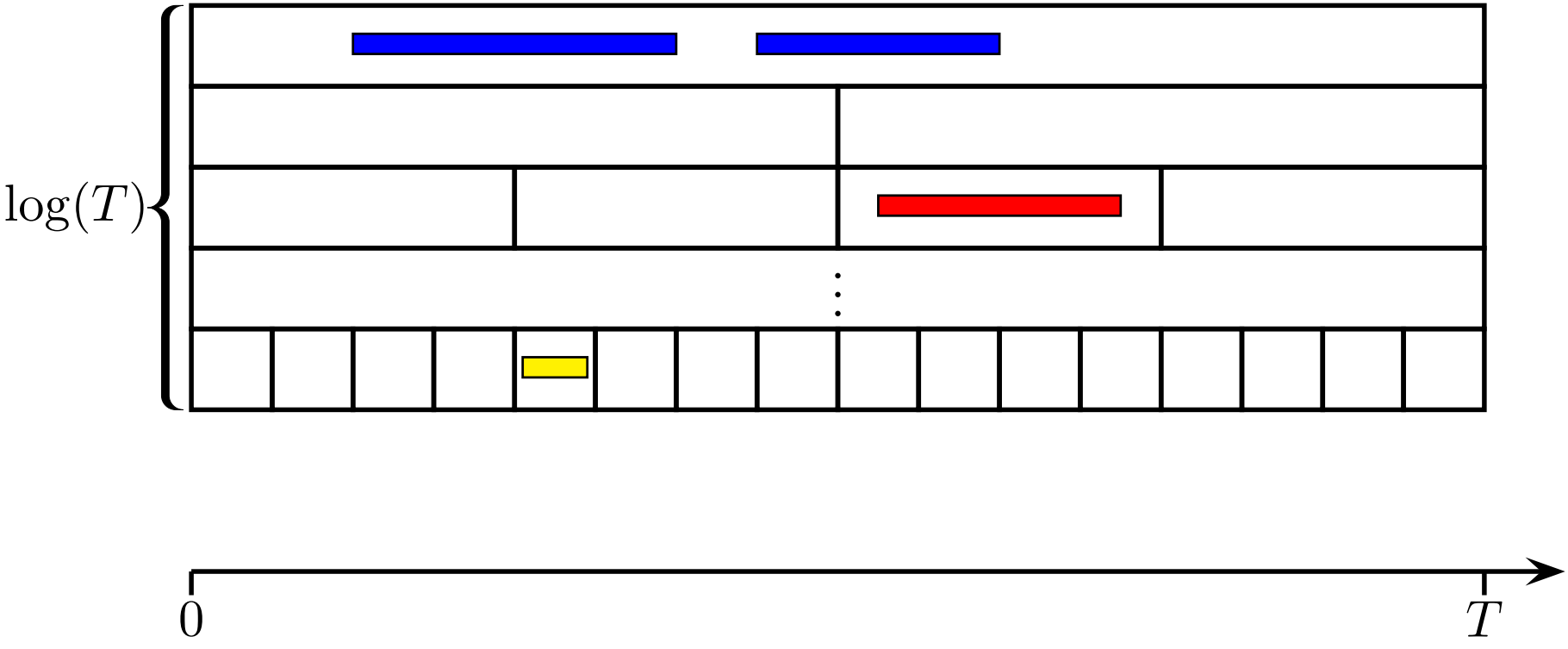
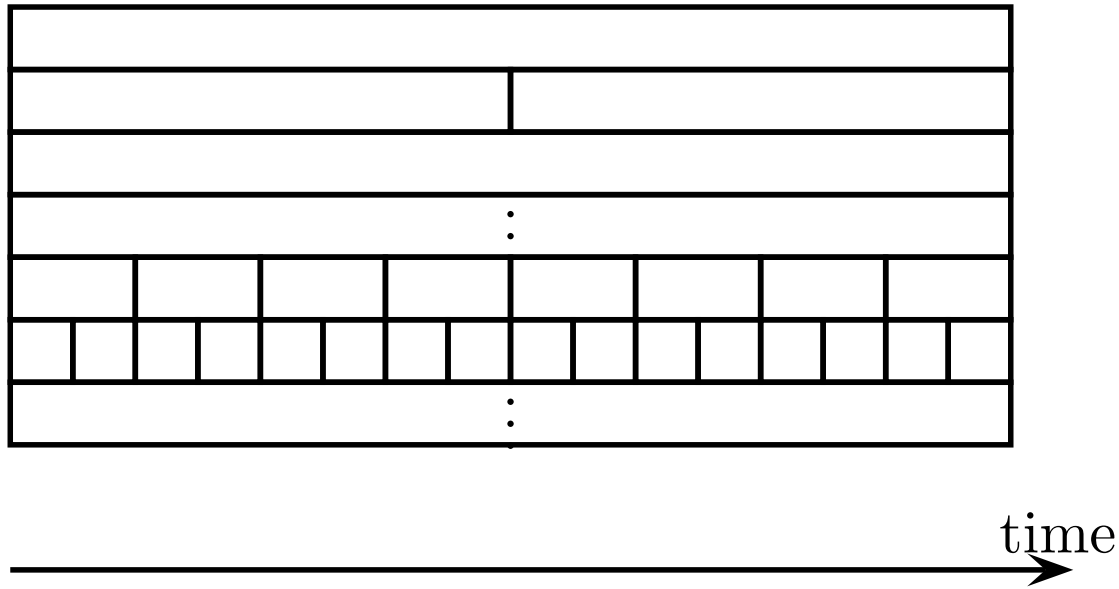


- Partition time horizon into binary family
- Assign jobs to min interval containing support

The algorithm



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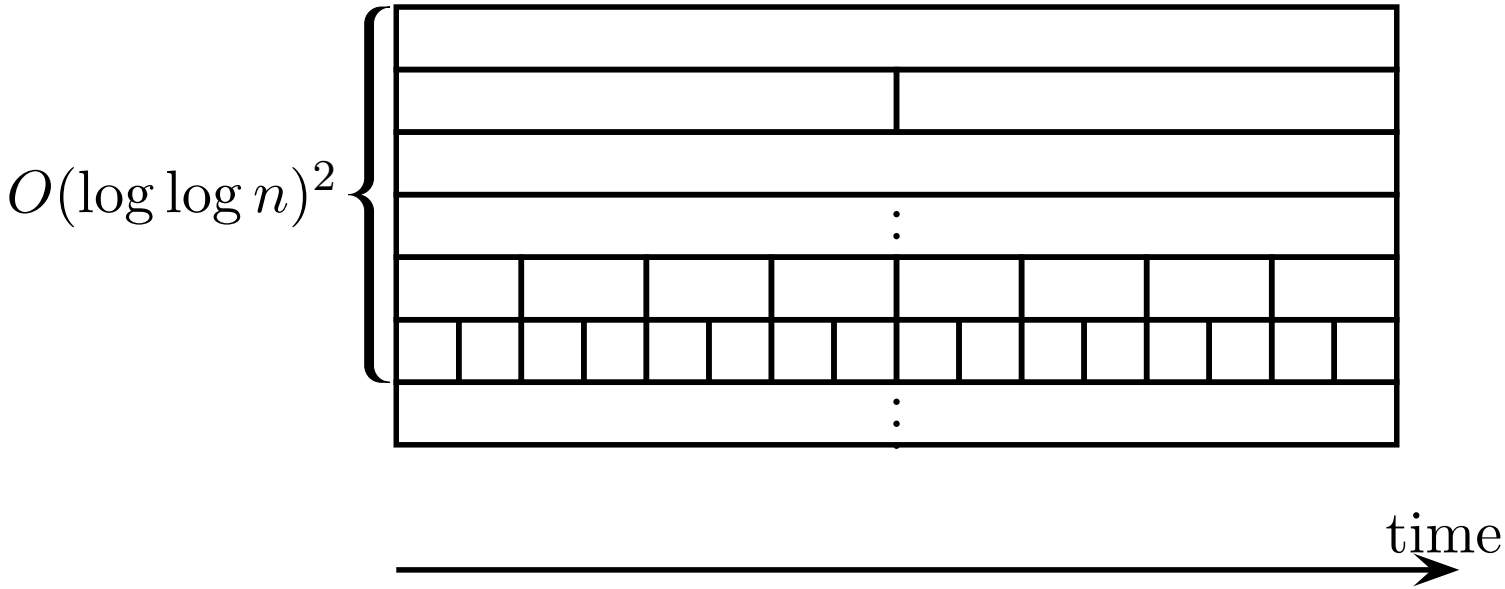
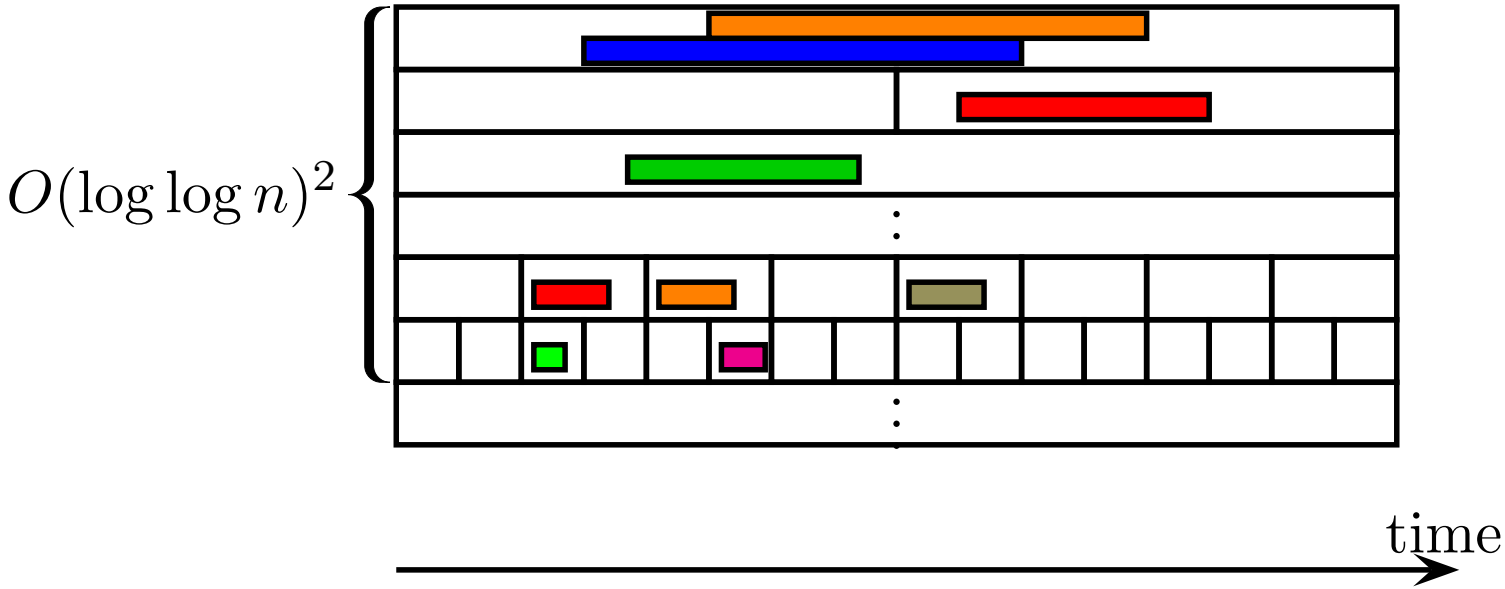


(1) Apply **conditioning** until max chain length of jobs in first $O(\log \log n)^2$ levels down to $\varepsilon' T$

The algorithm



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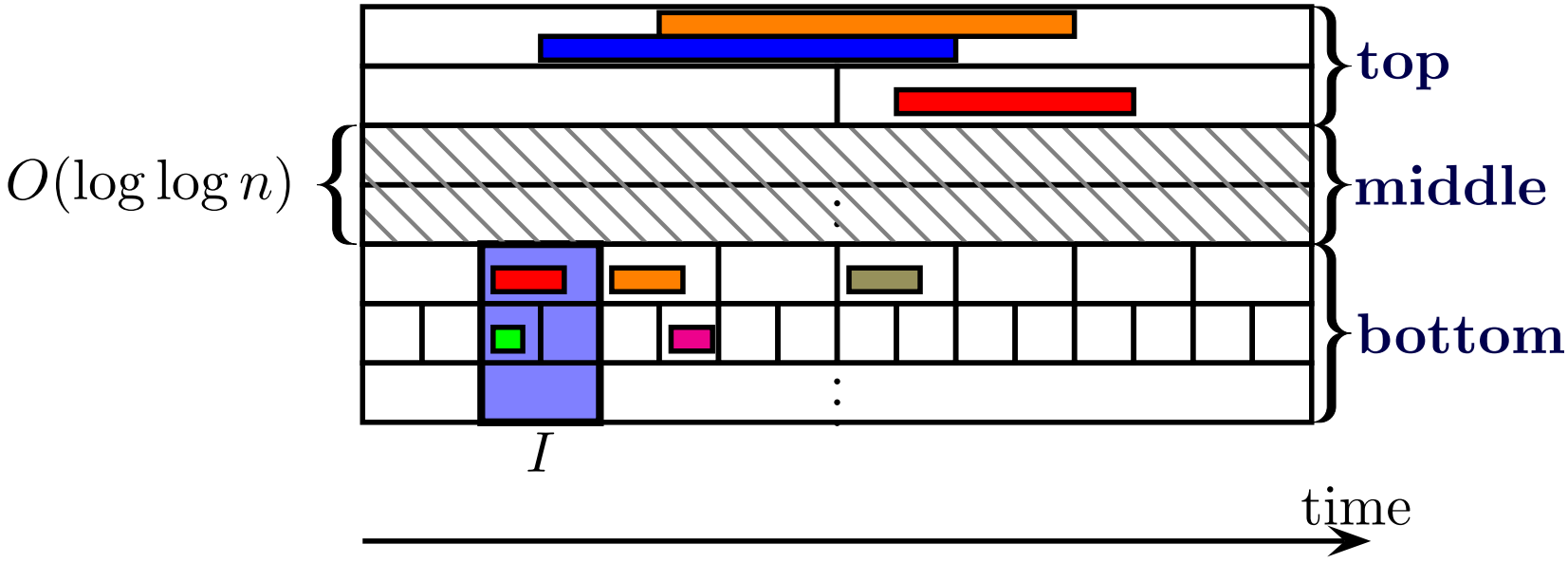

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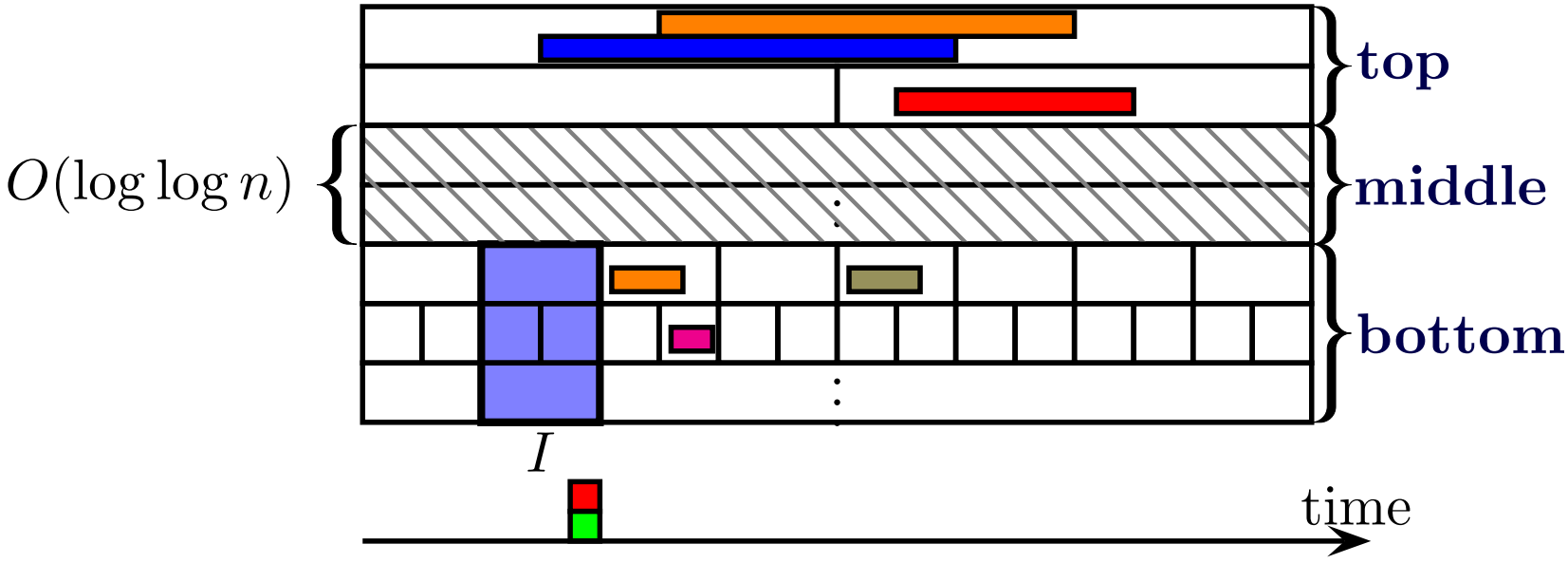

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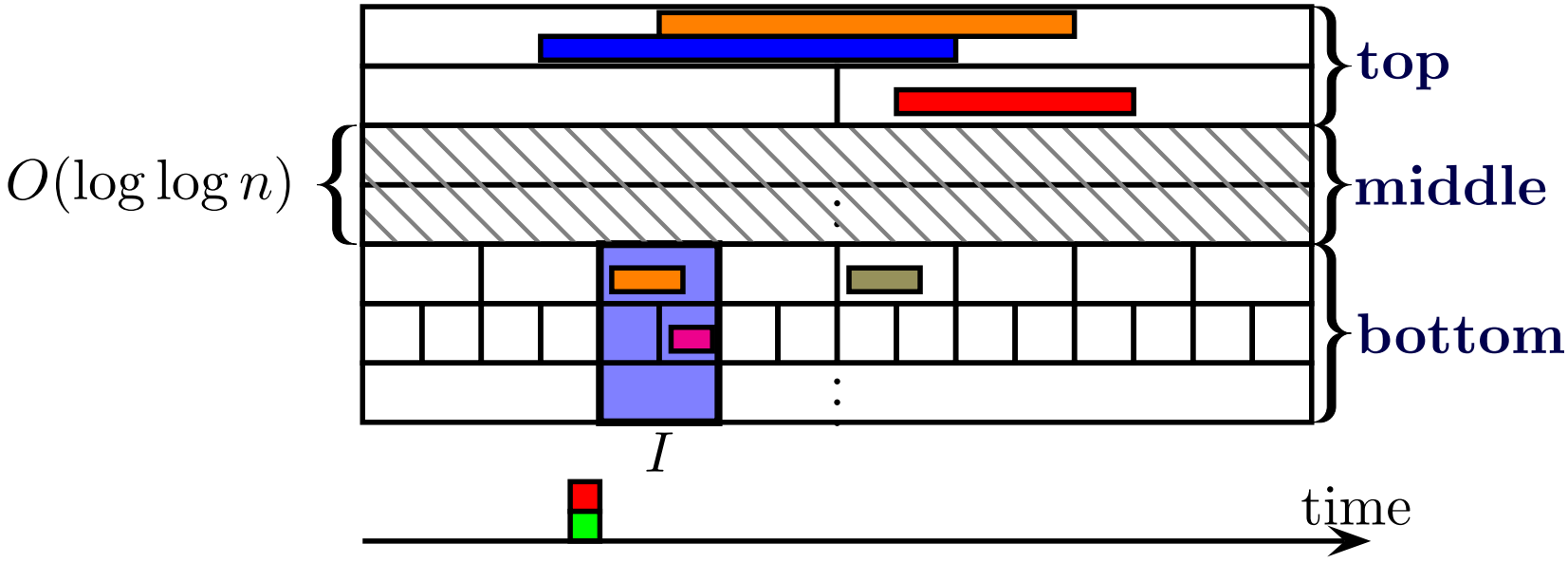

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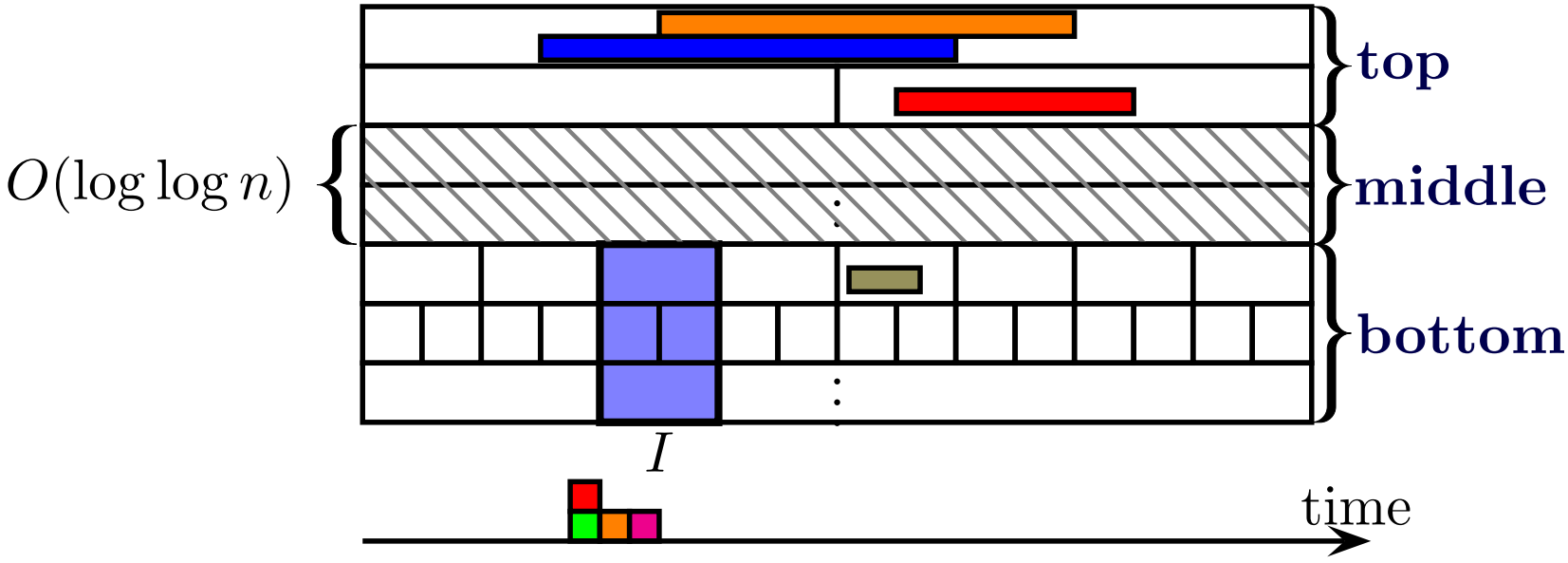

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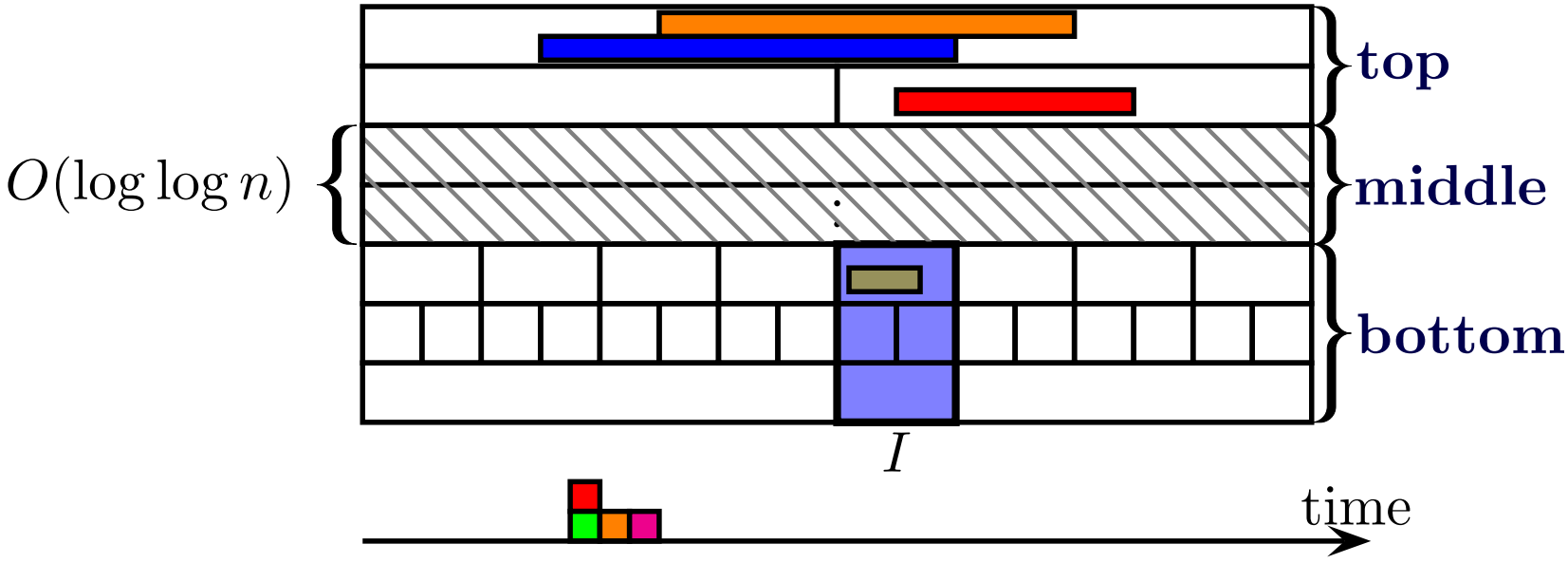

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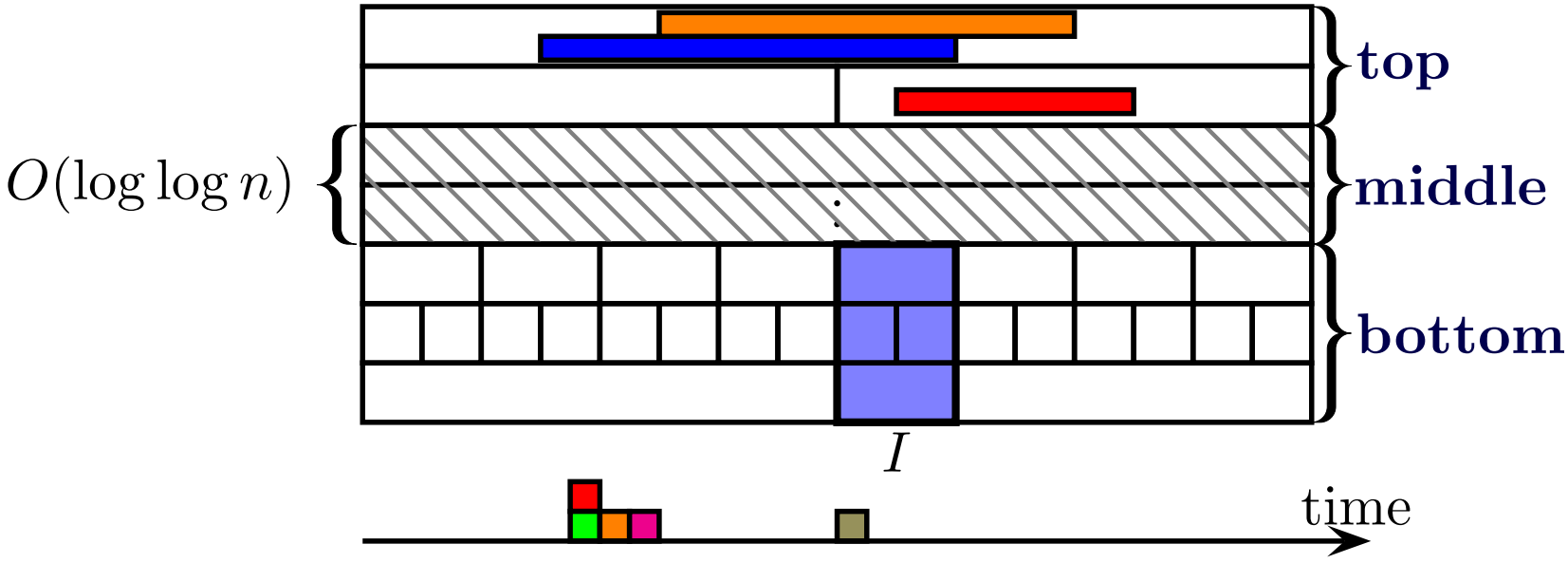

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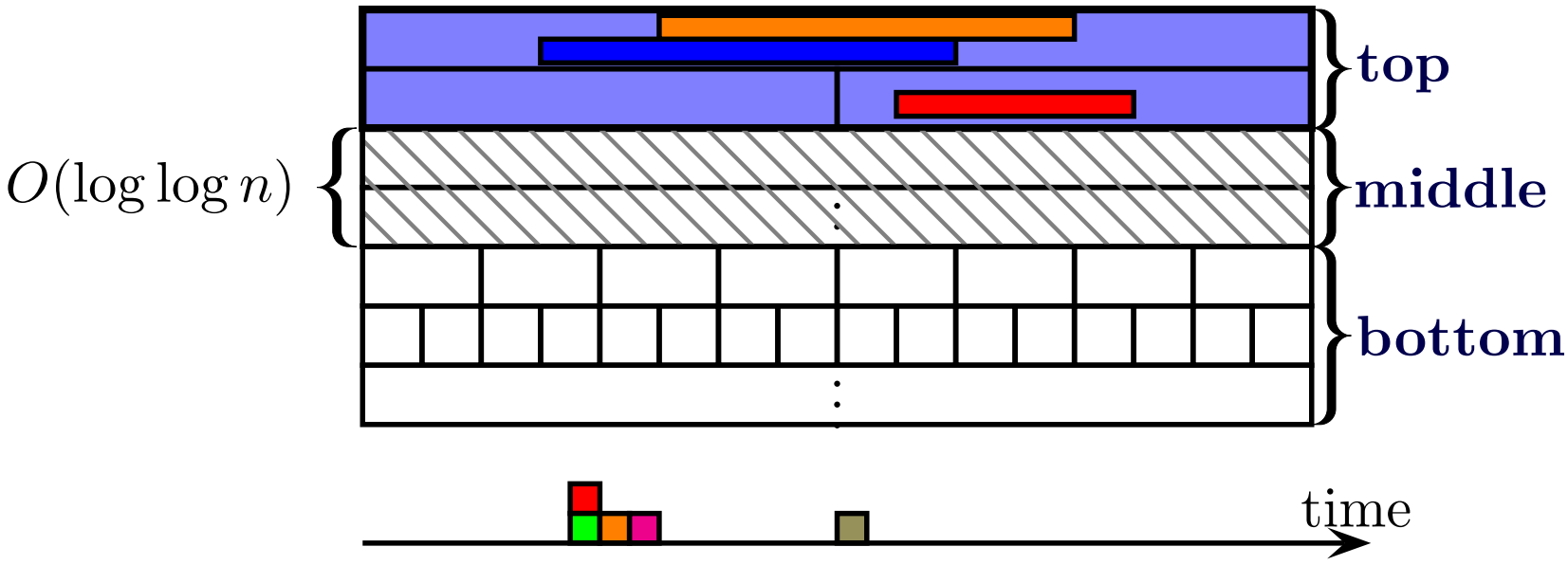

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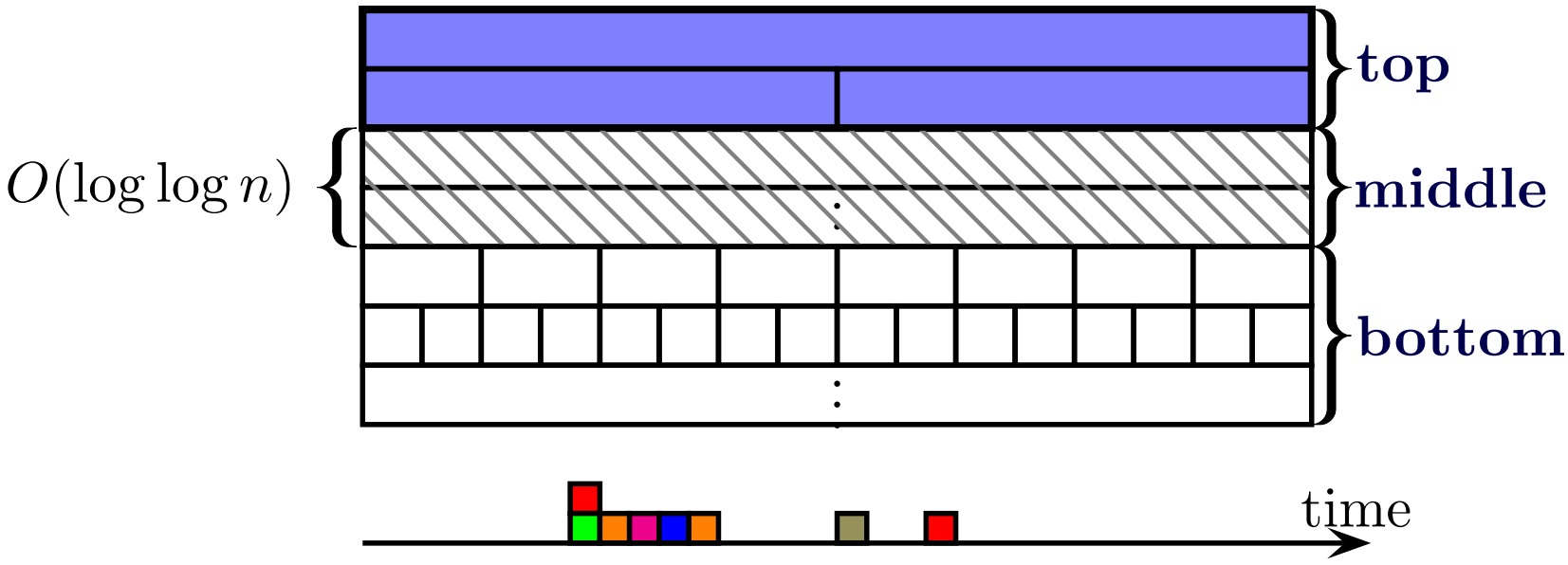

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After conditioning $2^{O(\log \log n)^2} \cdot \frac{1}{\varepsilon'}$ times, **maximum length chain** among top jobs is $\leq \varepsilon' \cdot T$.

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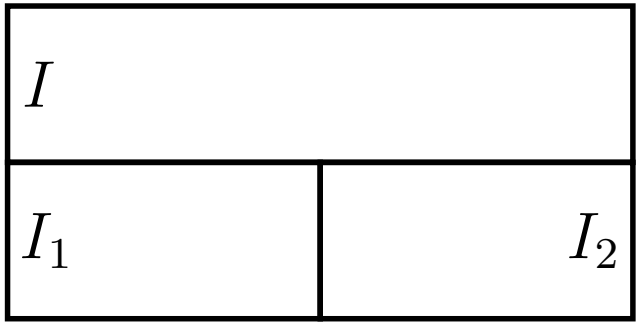

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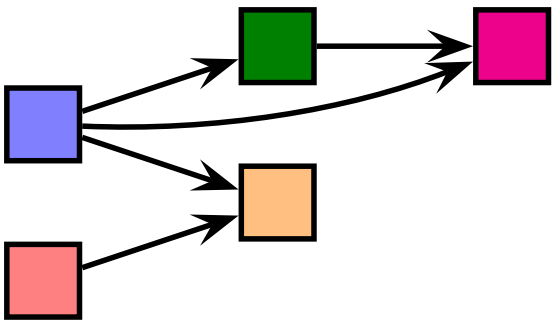

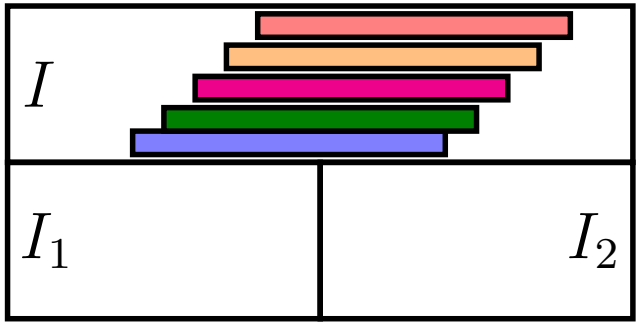

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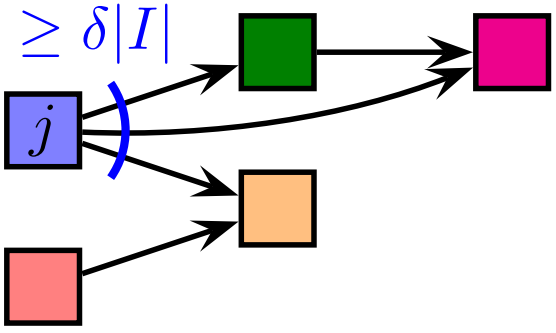

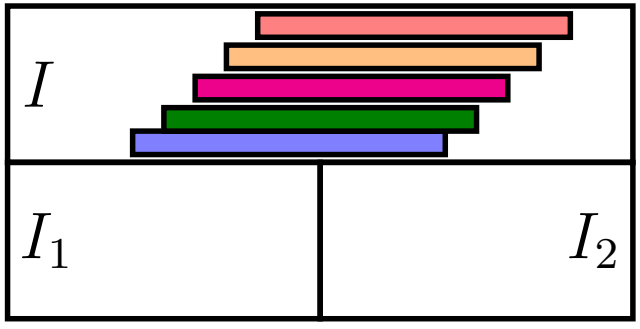

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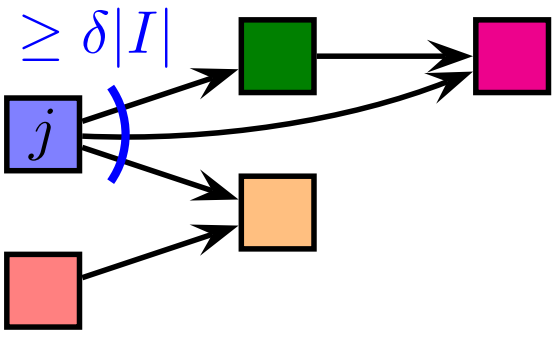

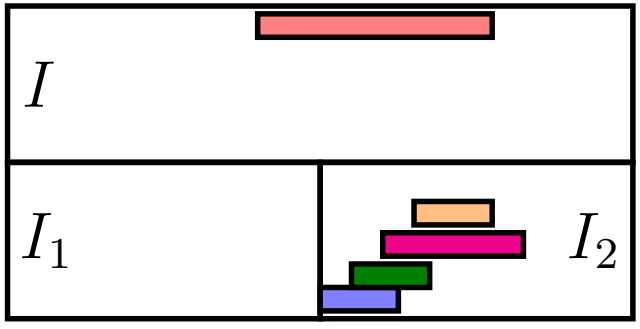

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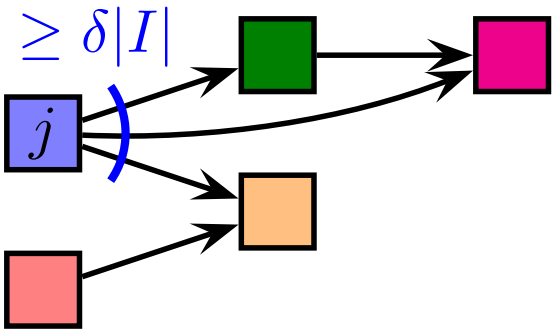

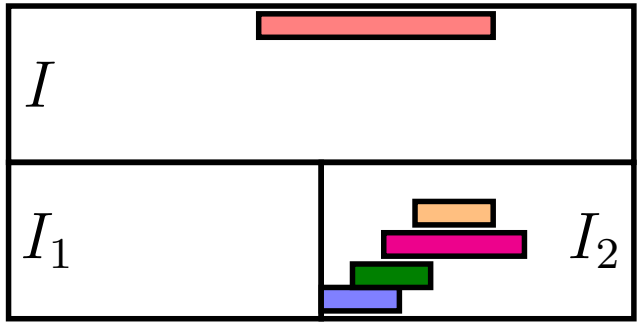

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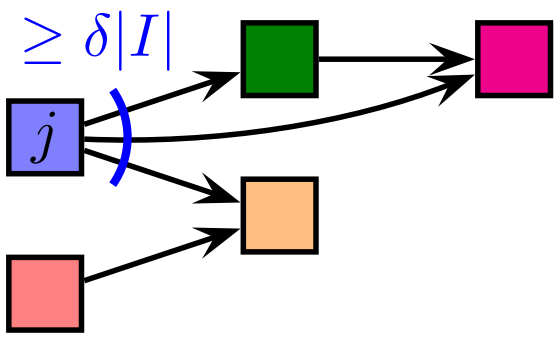

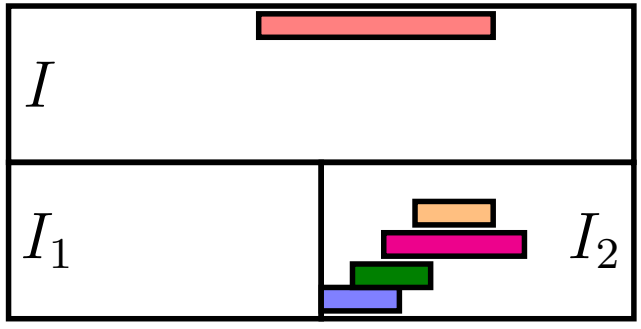

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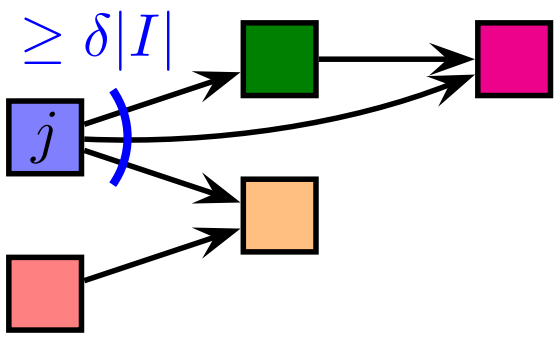
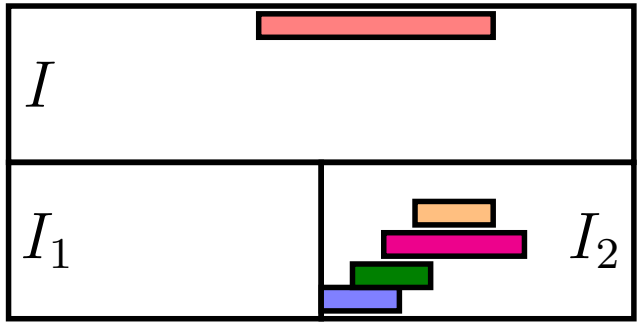


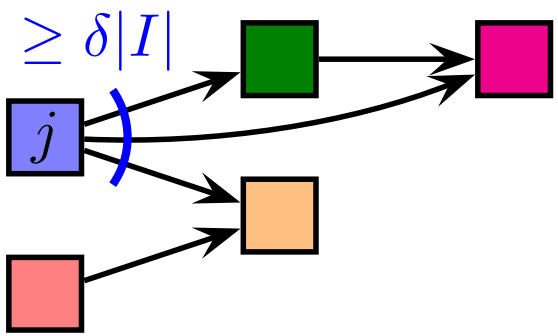


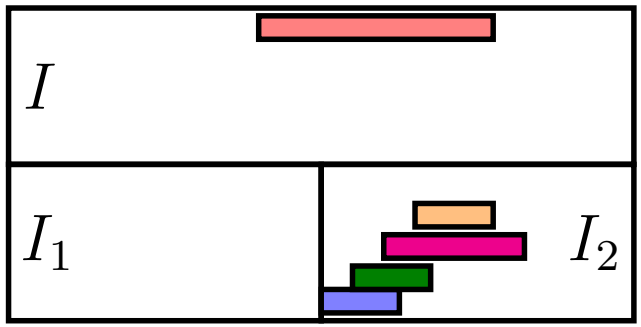
- Pick an interval I and consider jobs assigned to it.
- Consider j with $\delta|I|$ successors $\Rightarrow$ condition on $x_{j,I_2} = 1$
- Needed $\leq \frac{1}{\delta}$ times per interval; $2^{O(\log\log n)^2}$ intervals
- Loose $\leq \frac{1}{\delta} \cdot 2^{O(\log\log n)^2}$ Lasserre-rounds

Reducing chain length

Lemma

After conditioning $2^{O(\log\log n)^2} \cdot \frac{1}{\varepsilon'}$ times, **maximum length chain** among top jobs is $\leq \varepsilon' \cdot T$.





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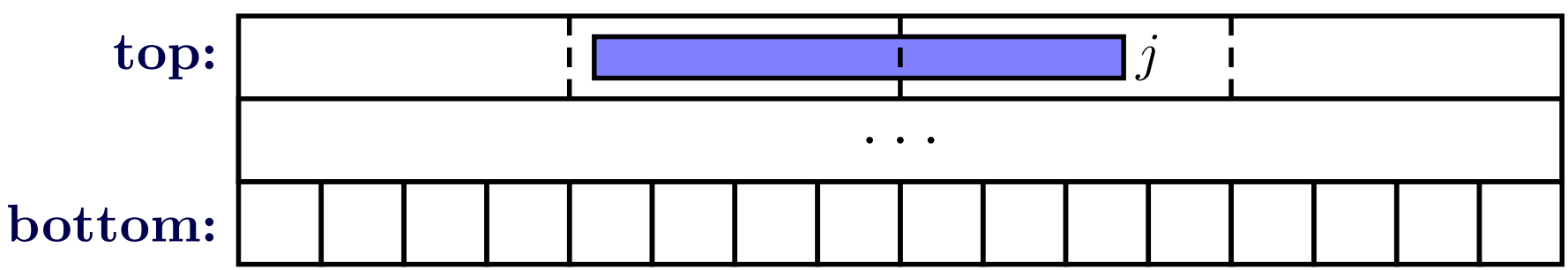
- Longest chain in top jobs: $\delta T \cdot O(\log\log n)^2$ □

Scheduling top jobs (I)



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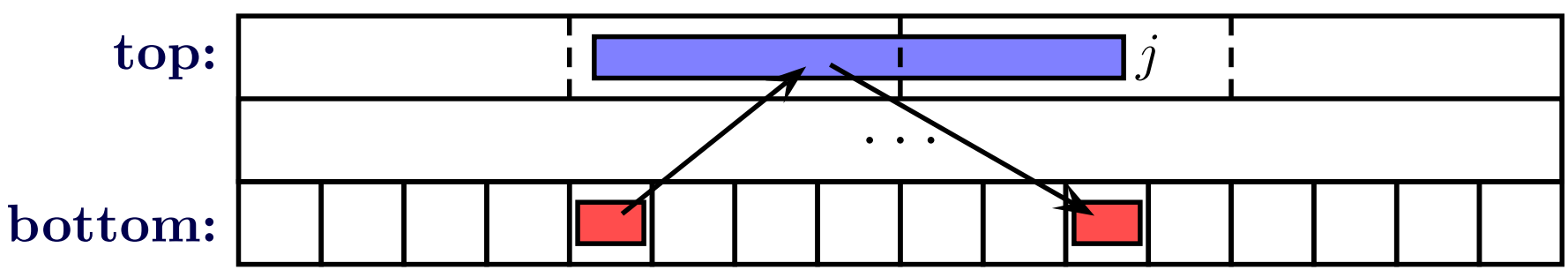
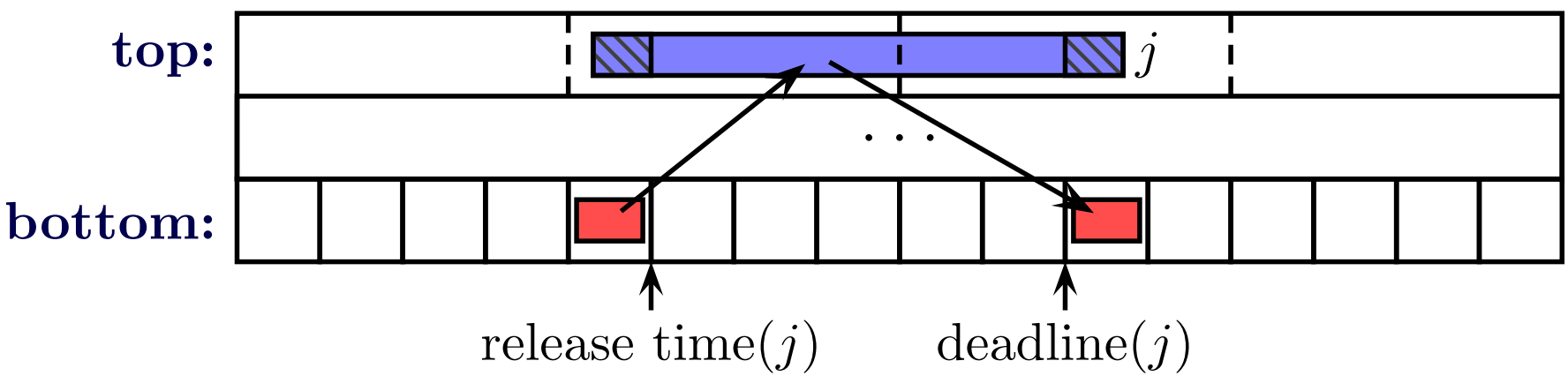


Scheduling top jobs (I)

- For top job j set
  $[\text{release time}(j), \text{deadline}(j)] :=$ fully contained bottom intervals



Scheduling top jobs (I)

- For top job j set
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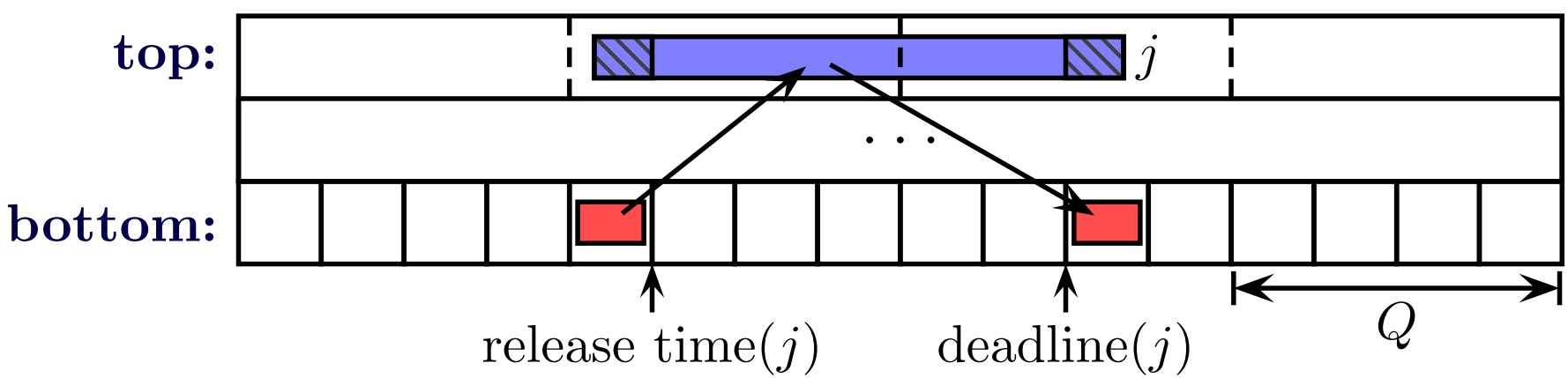
- Set $Q := \text{polylog}(n)$ as top/bottom gap



Scheduling top jobs (I)

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Can assign top jobs respecting release times/deadlines, ignoring prec. constraints & discarding $\frac{2Tm}{Q}$ jobs.

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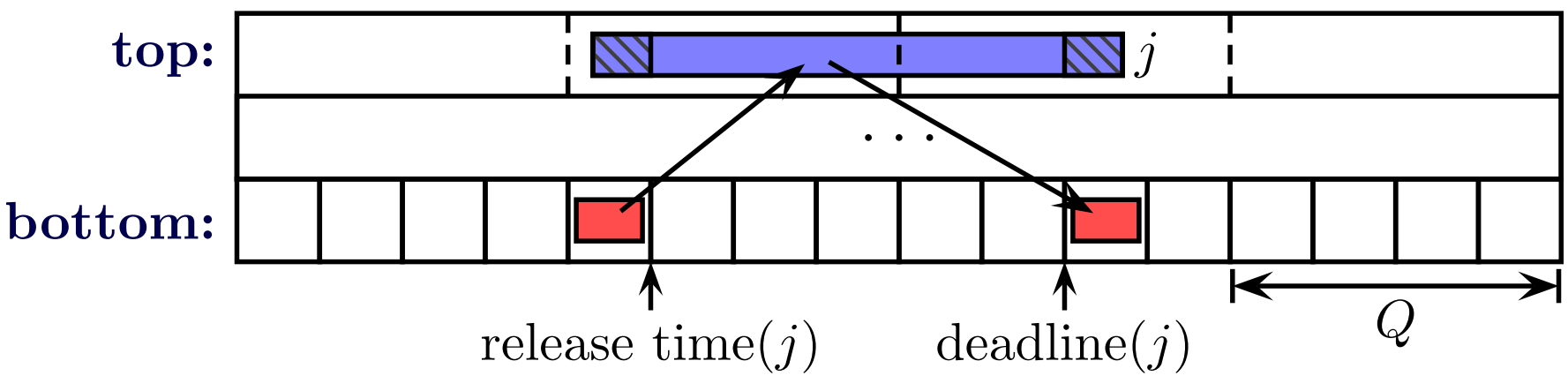

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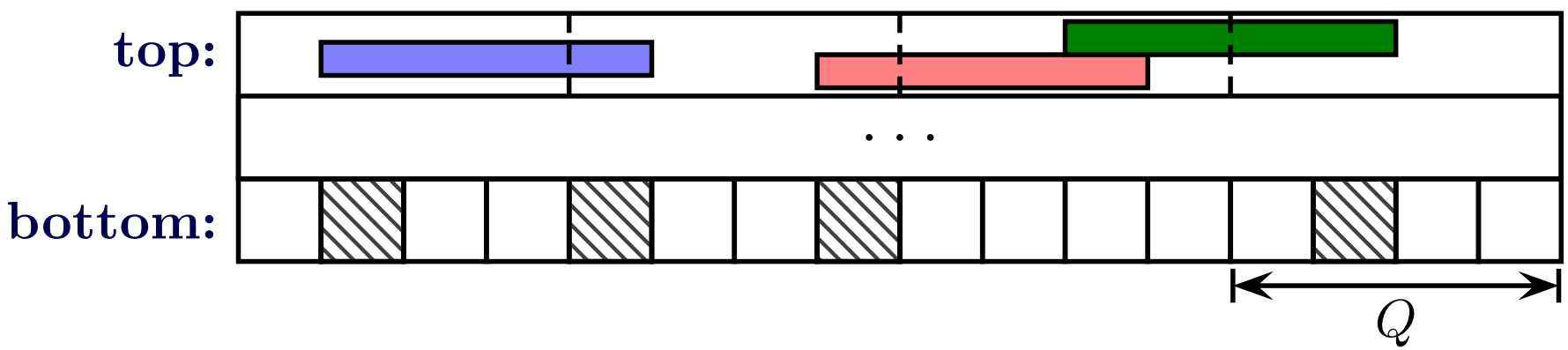
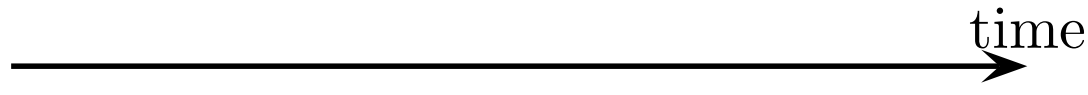
- **Hall's Theorem:** Loss for jobs J' is $\frac{2}{Q}$-fraction of slots



EDF (1)

Setting:



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Setting:

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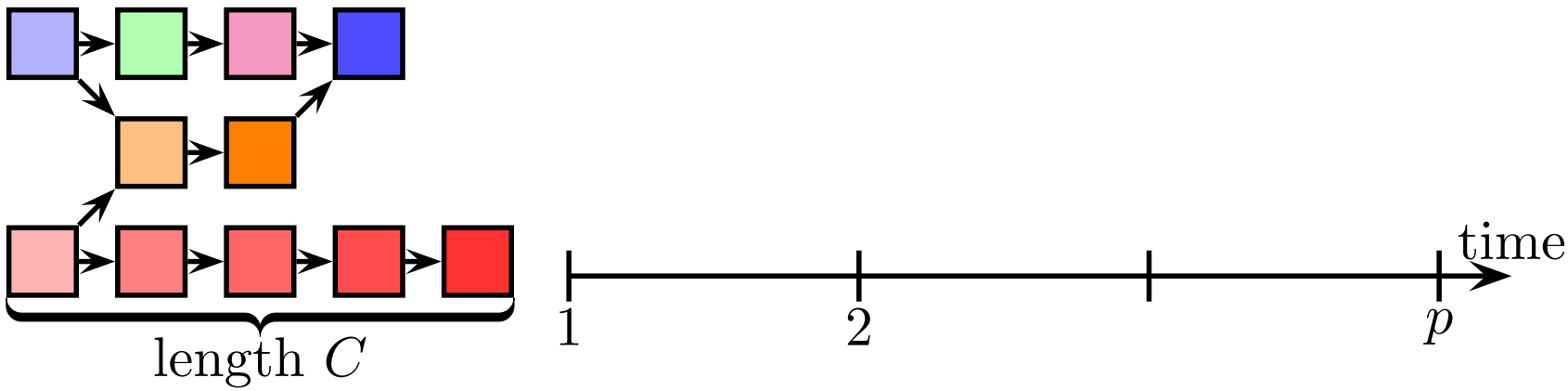

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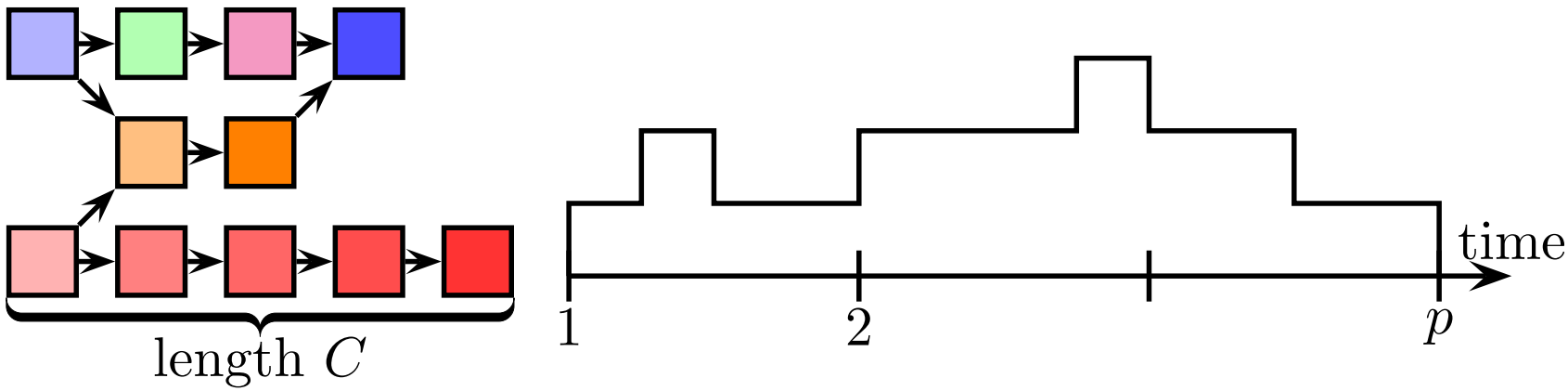

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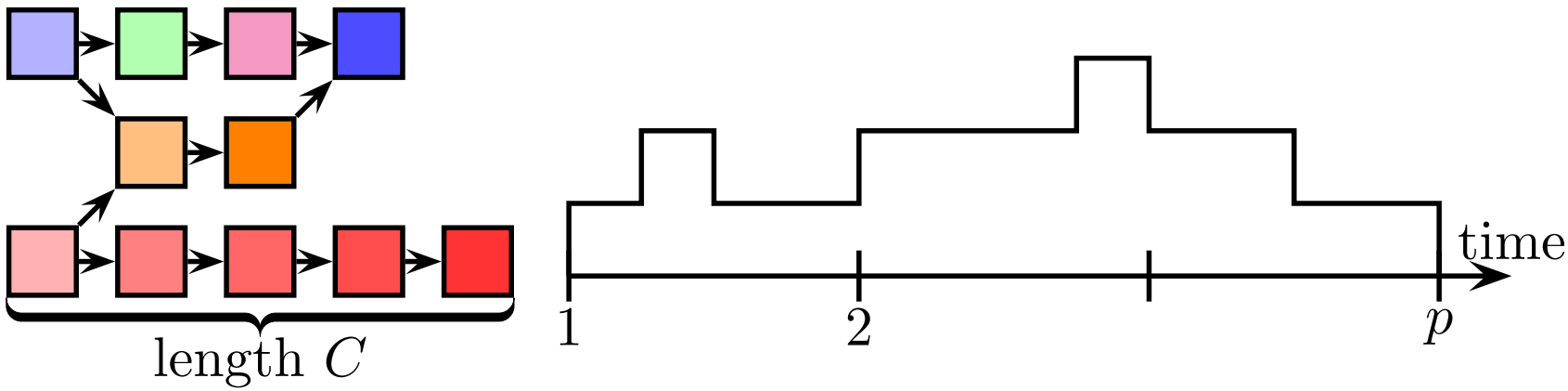

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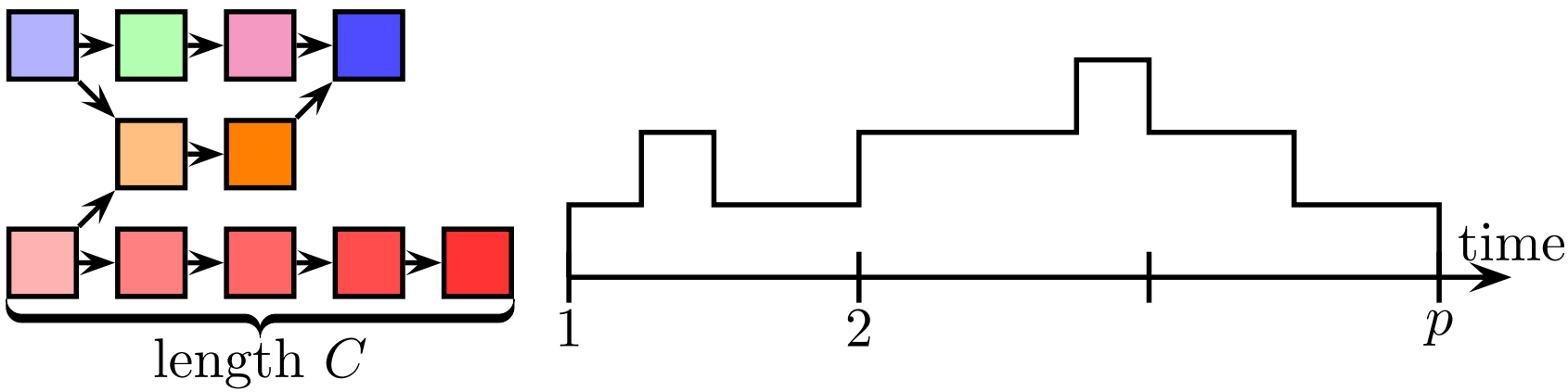


Earliest Deadline First:

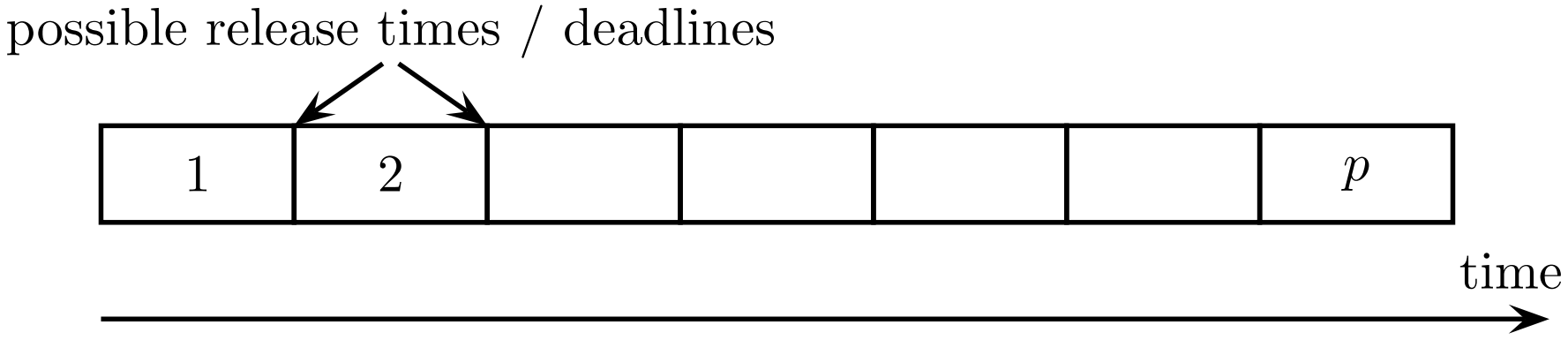
(1) FOR $t = 1$ TO T DO

(2) Process available job with minimum deadline

(3) If any job not done by deadline discard it

EDF (2)

Analysis:



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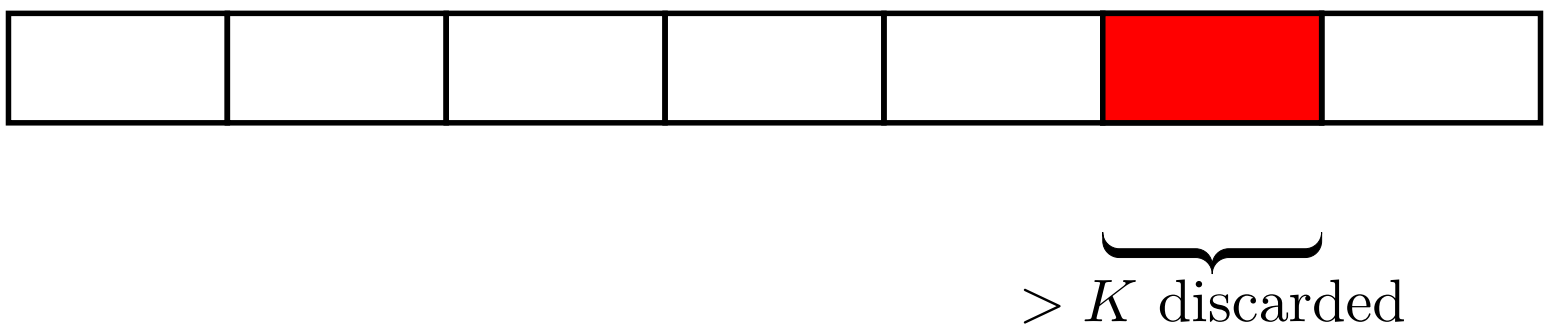

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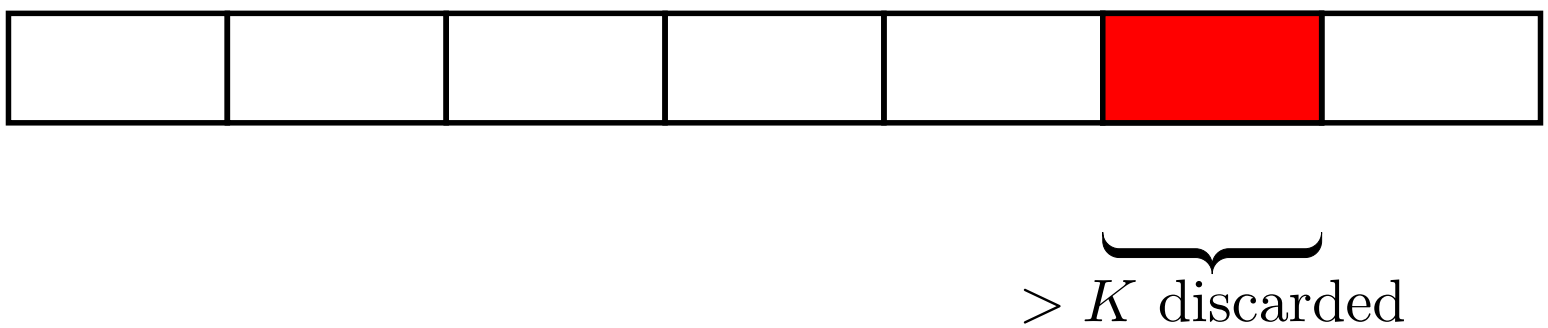

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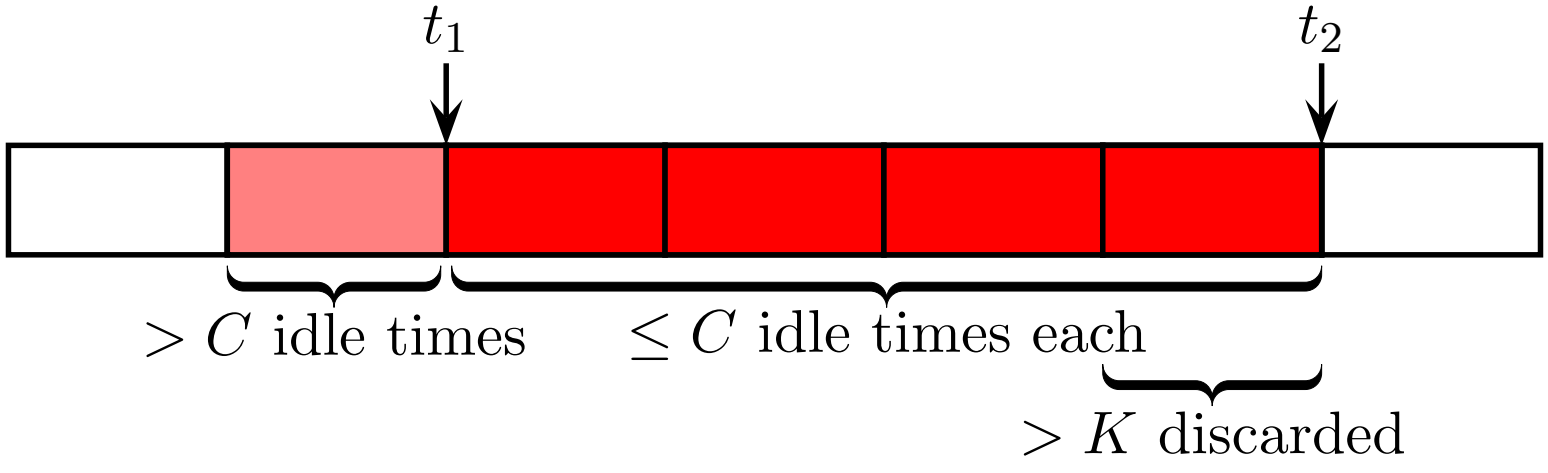

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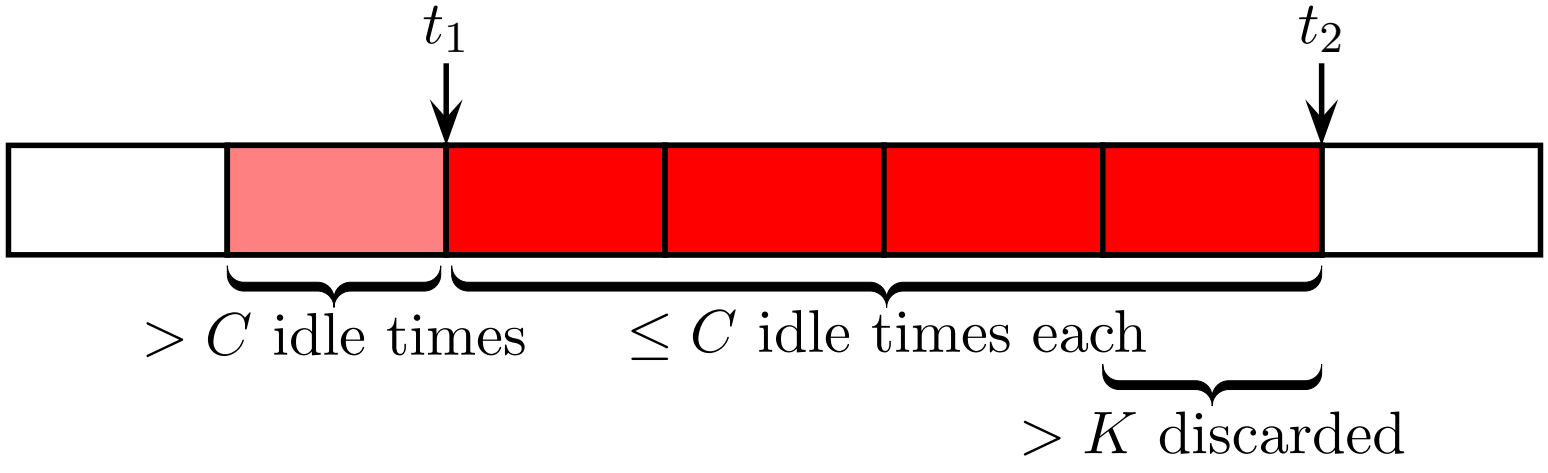

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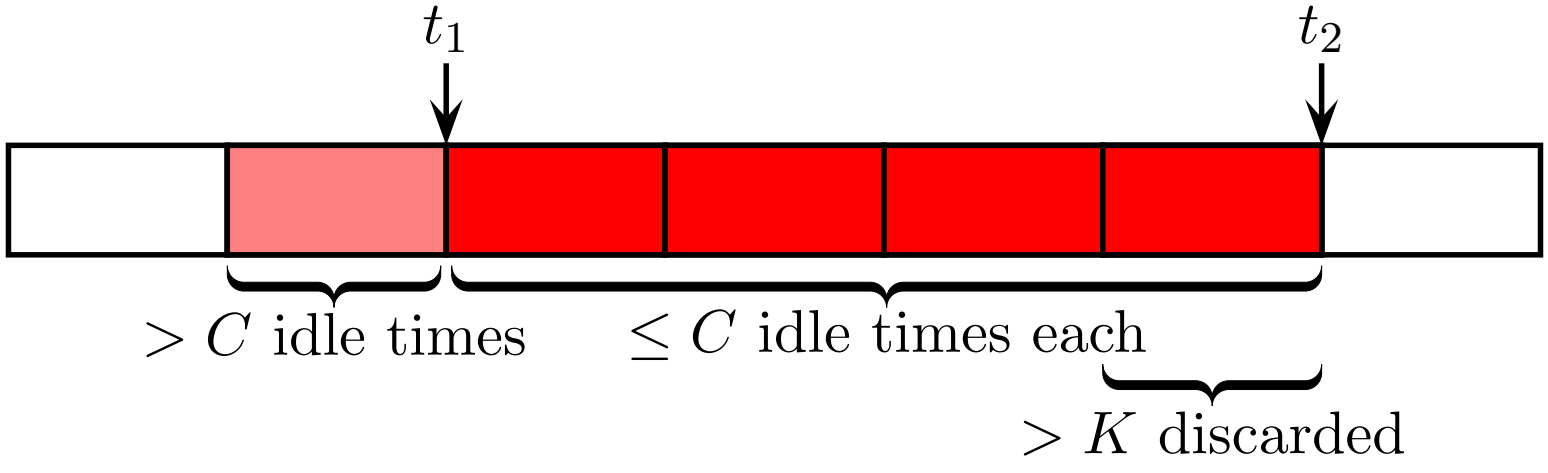
- $K \leq$ idle slots in $[t_1, t_2] \leq pmC$ (feasibility w/o prec.-constr.)



The end

Open problems:

- Are $c(\varepsilon, m)$-Lasserre rounds enough?
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Thanks for your attention