Polynomiality for Bin Packing with a Constant Number of Item Types

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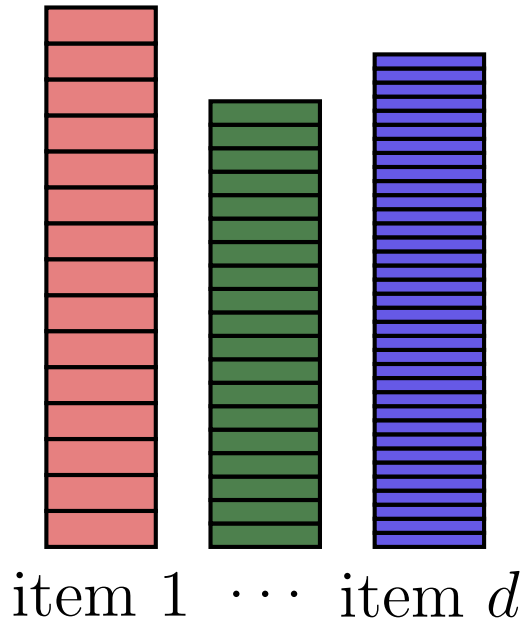
MIP 2013, Madison

Massachusetts
Institute of
Technology

Bin Packing / Cutting Stock

Input:



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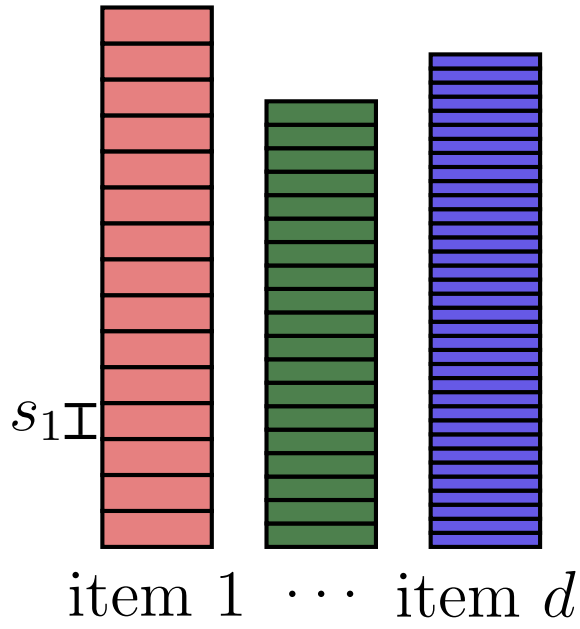

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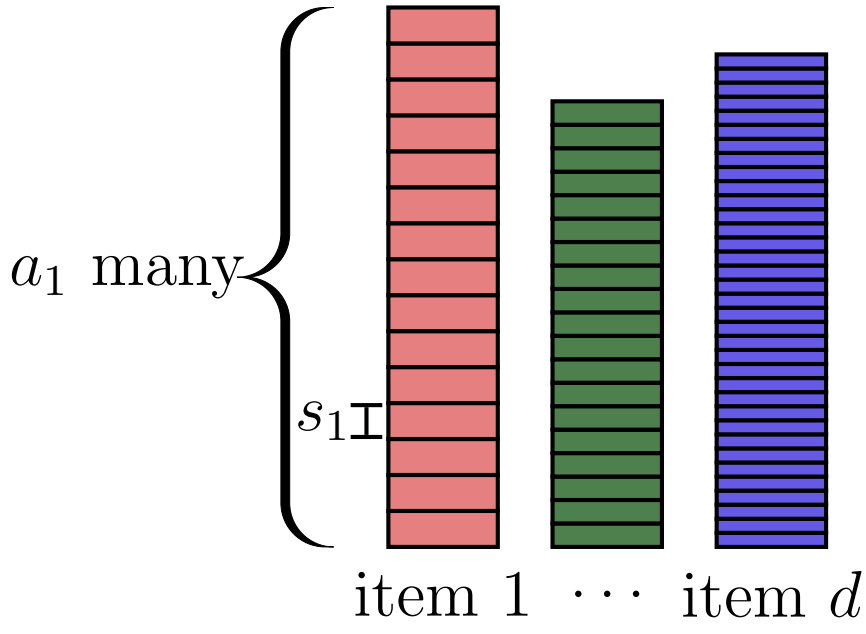

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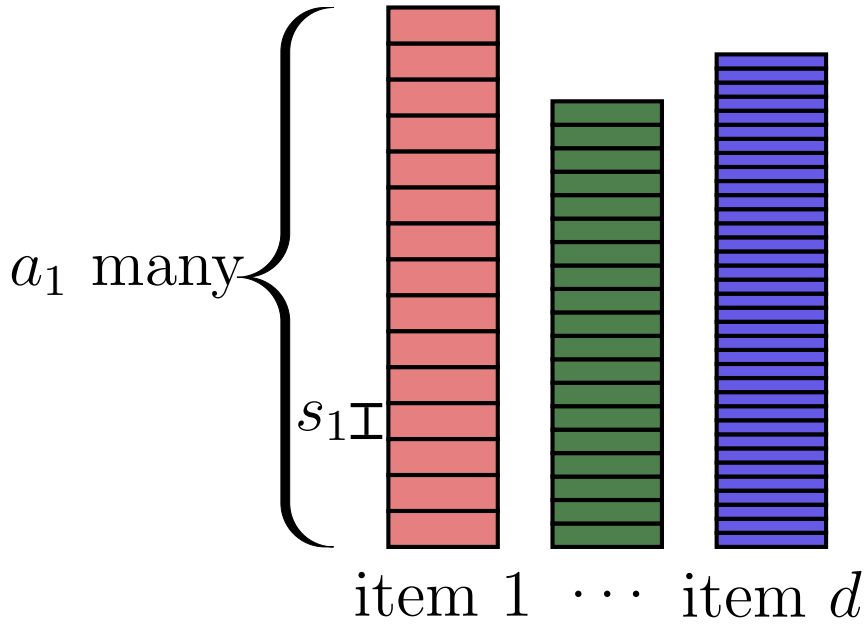

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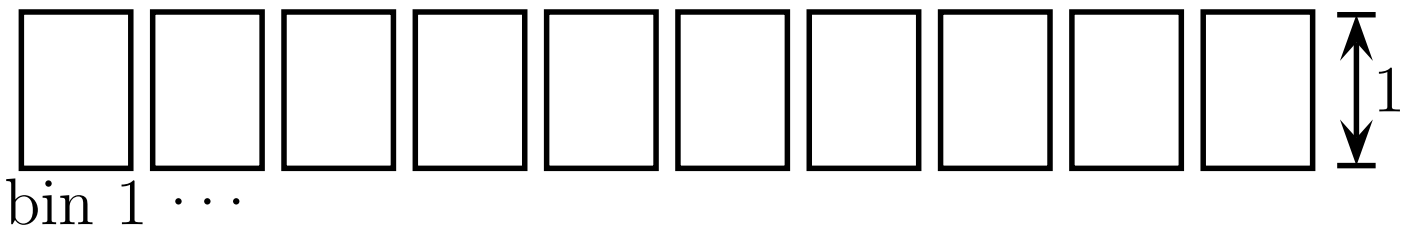

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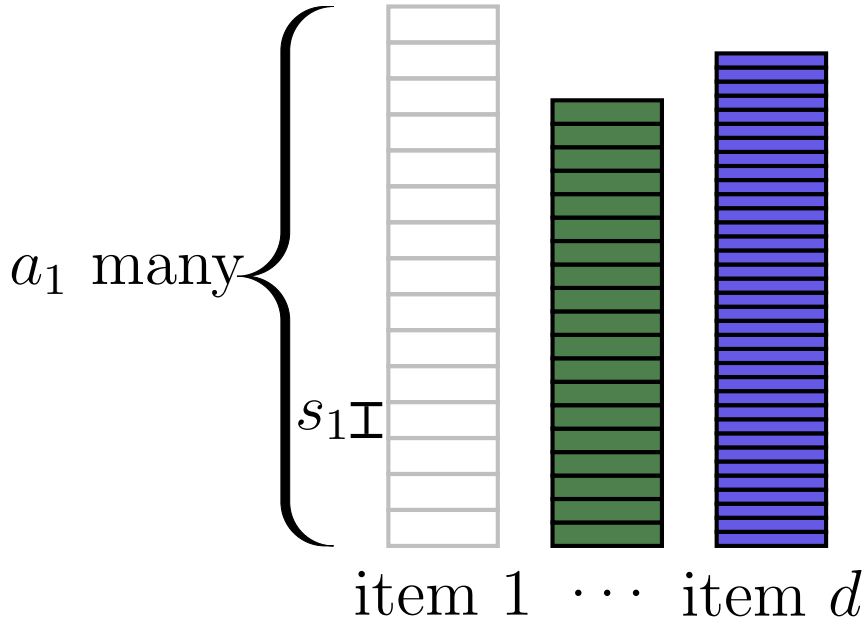

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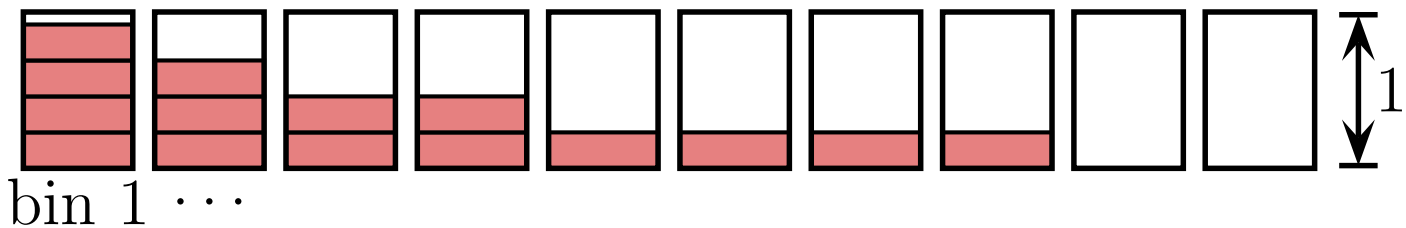

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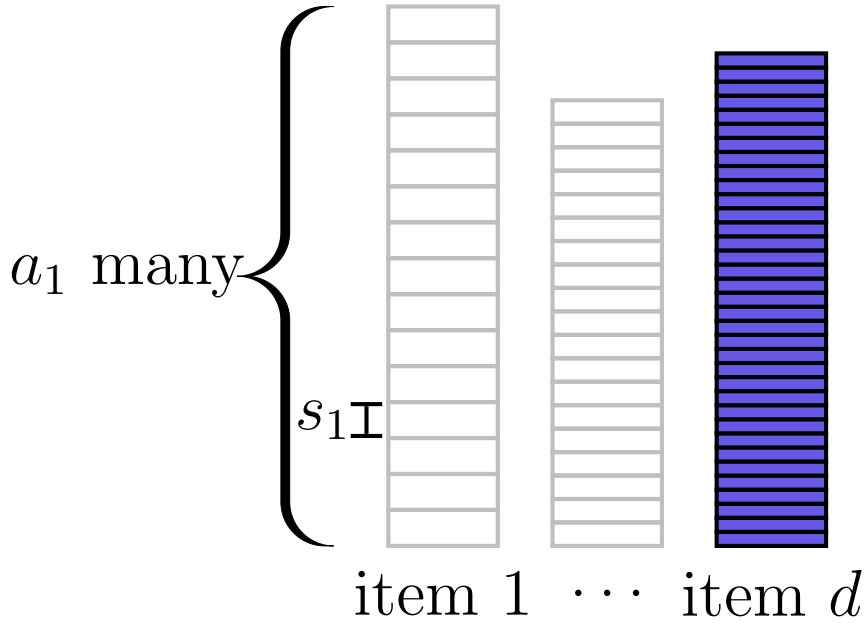

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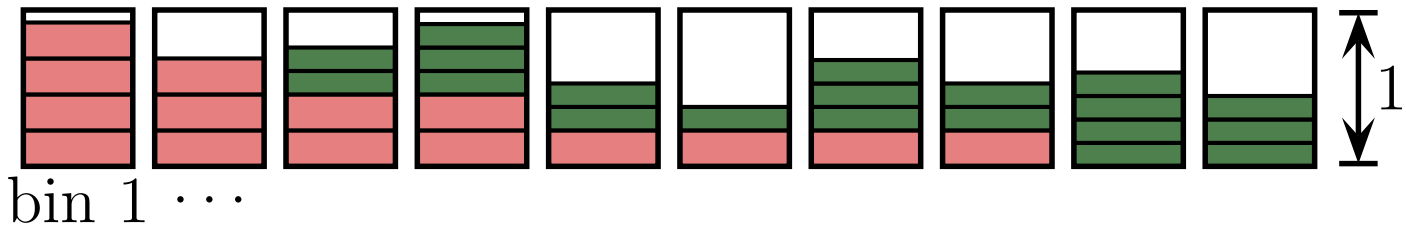

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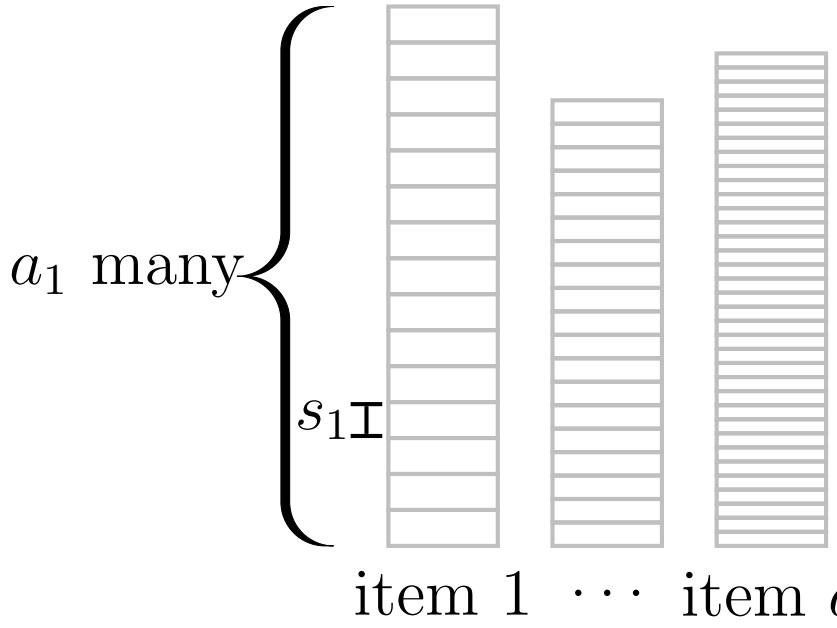

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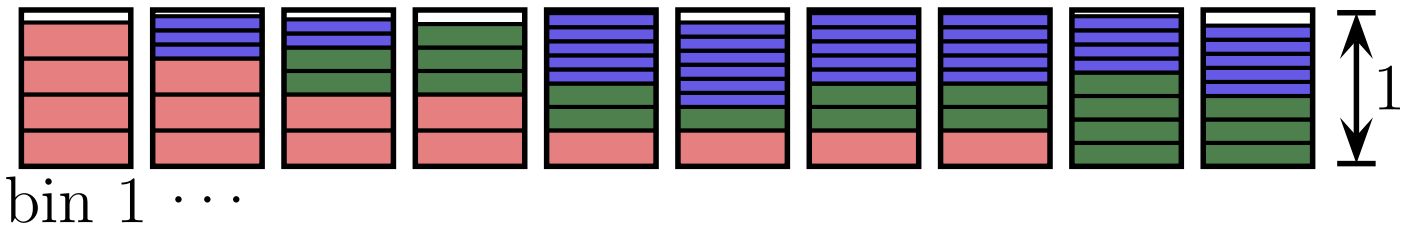

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Open problem [ES'06, MSS'97, F'07]

Solvable in poly-time for $d = 3$?

A geometric view

- Define $P = \{x \in \mathbb{R}^d_{\geq 0} \mid s^T x \leq 1\}$

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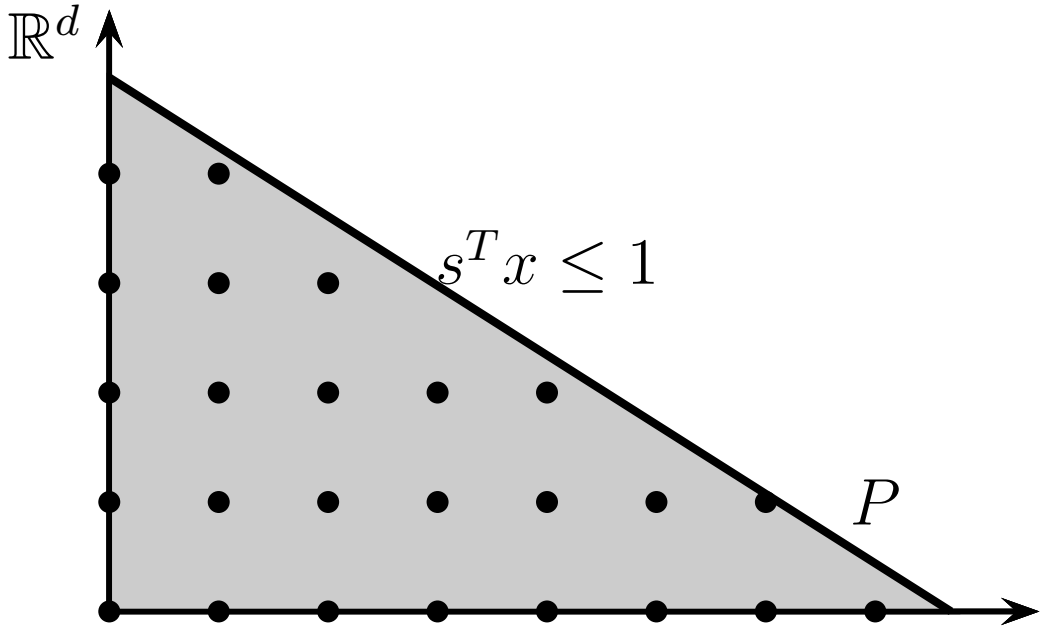

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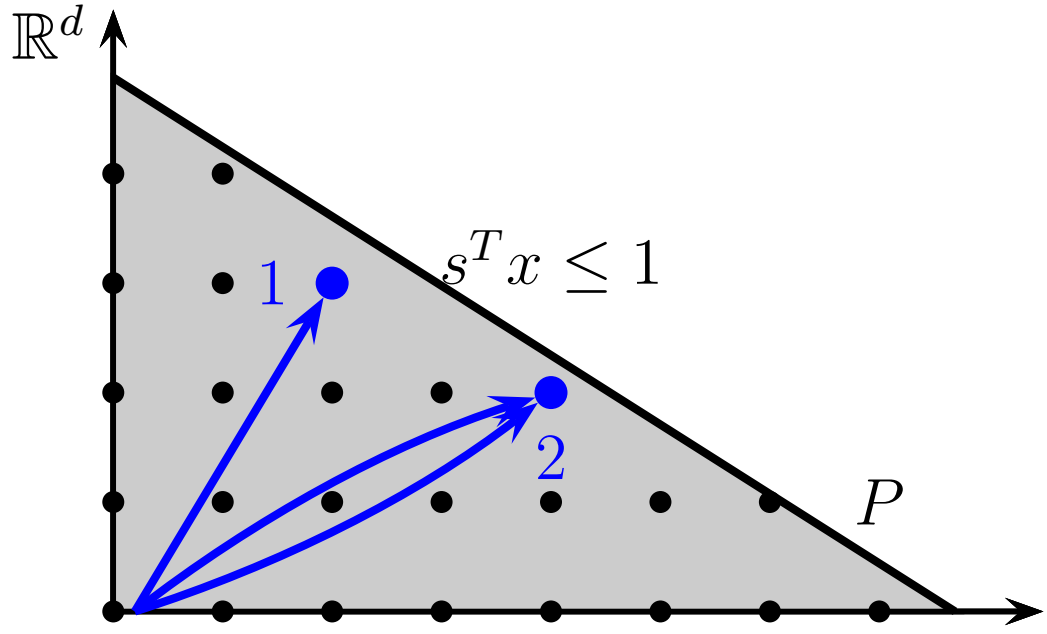

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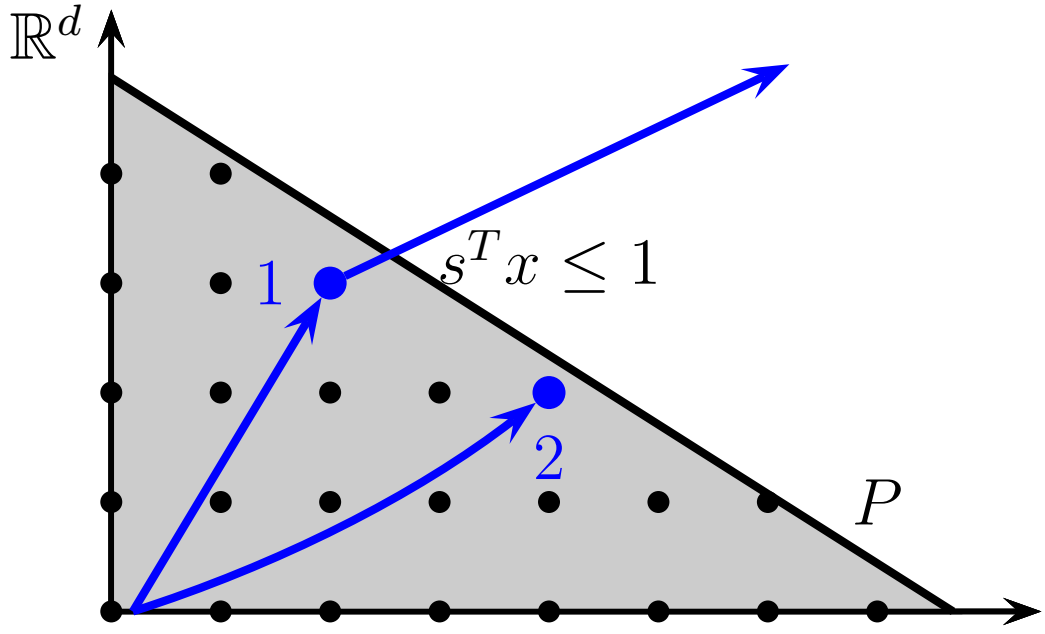

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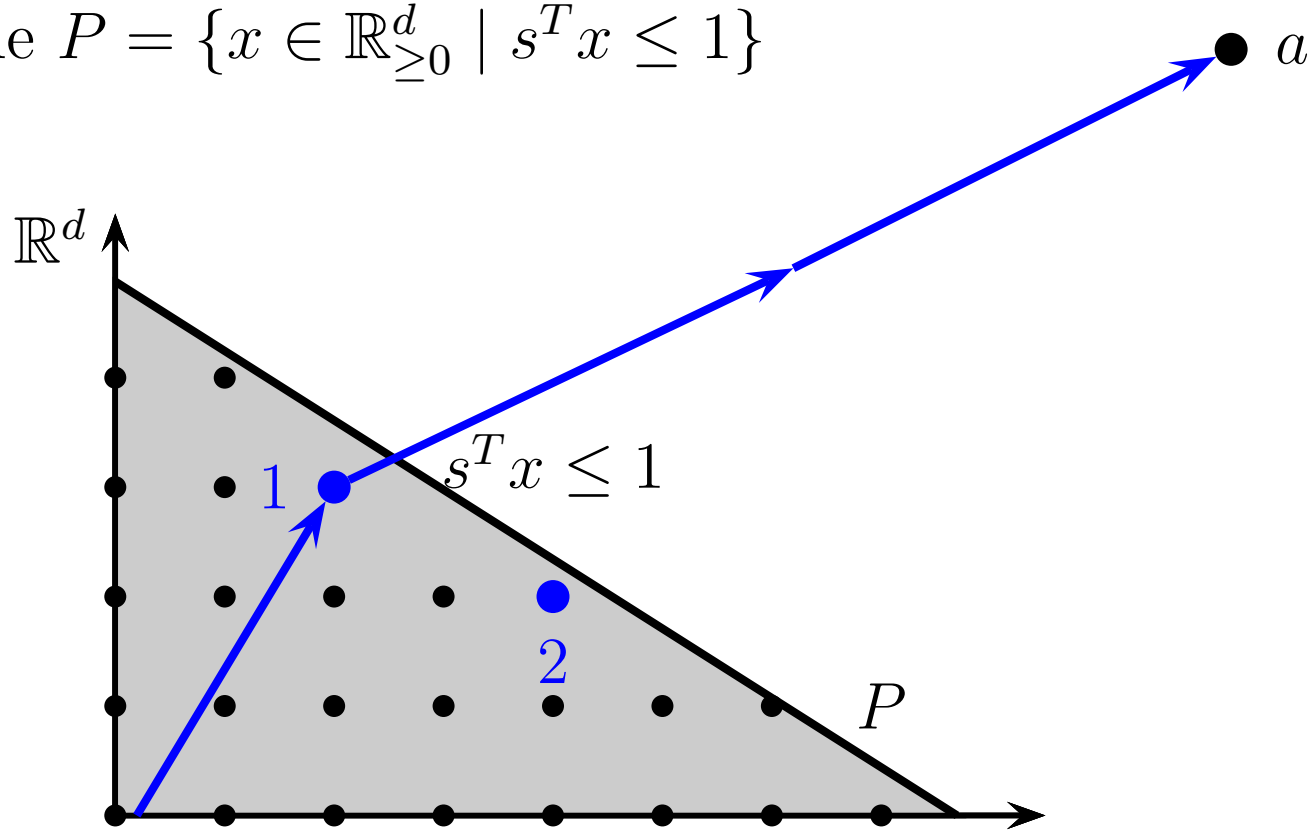

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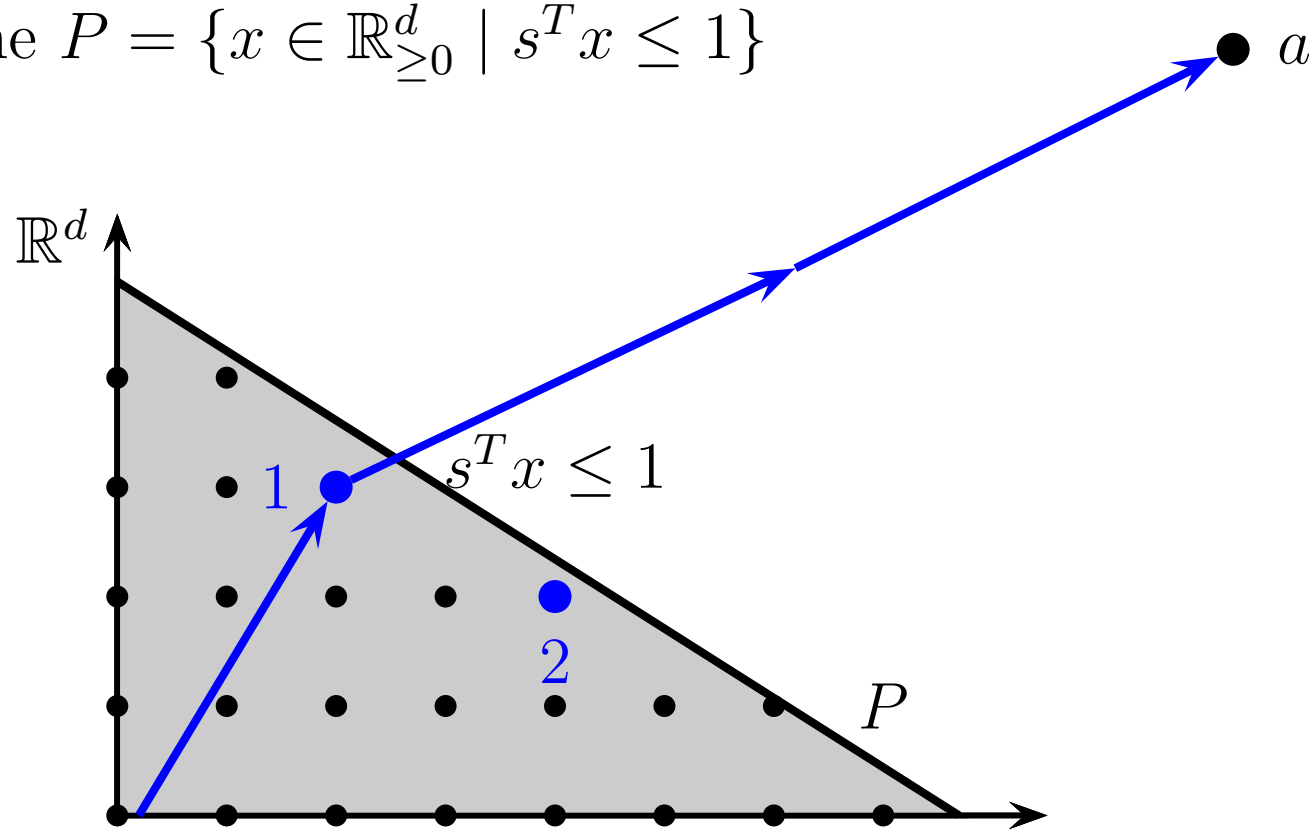

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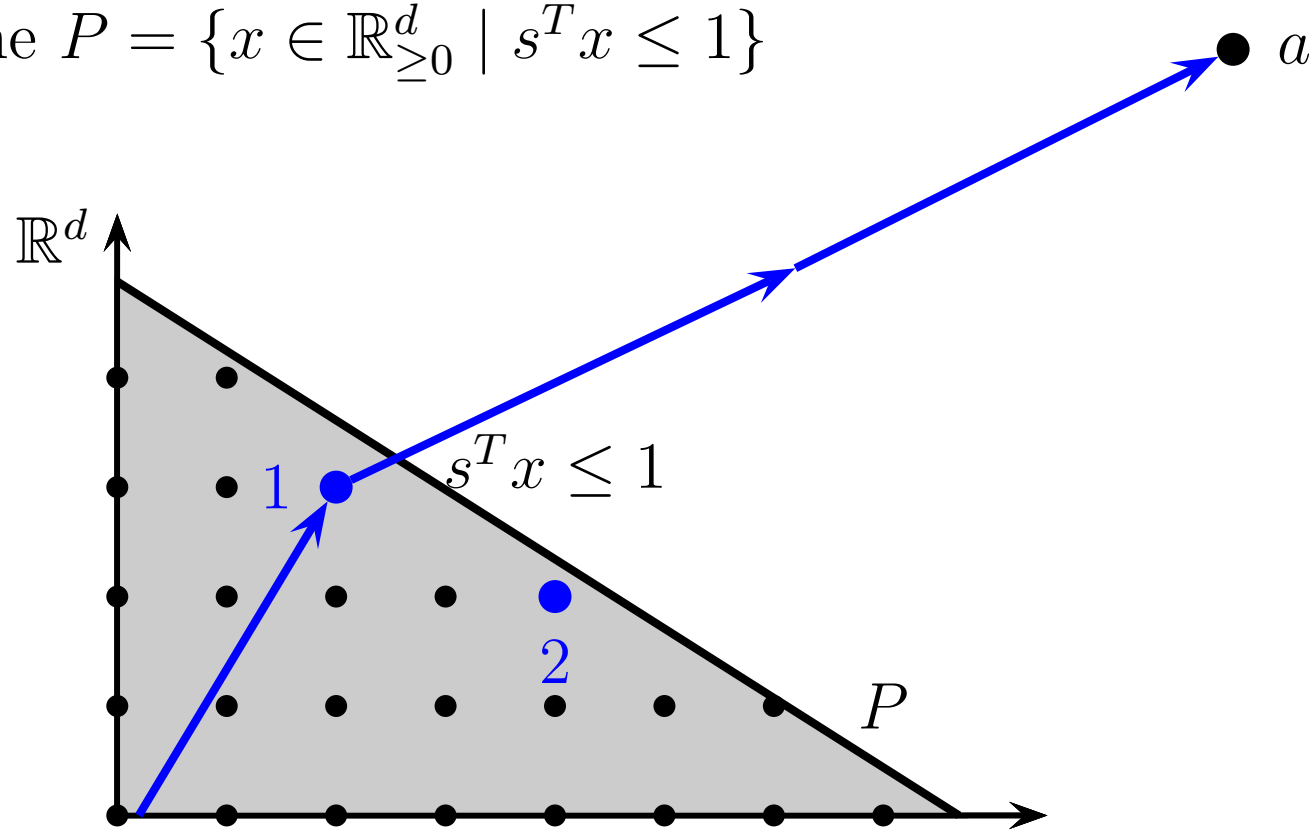

Problems:

- Points in P **exponentially** many
- Weights can be **exponential**

Main results

Theorem (Goemans, R. '13)

Bin Packing with $d = O(1)$ item sizes can be solved in **poly-time**.

Solves question by

- [McCormick, Smallwood, Spieksma '97]: "*might be* **NP**-*hard for* $d = 3$"
- [Eisenbrand & Shmonin '06]
- [Filippi '07]: "*hard open problem for general* d"

Main results (2)

- **Def.:** $\text{int.cone}(X) := \{\sum_{x \in X} \lambda_x \cdot x \mid \lambda_x \in \mathbb{Z}_{\geq 0}\}$

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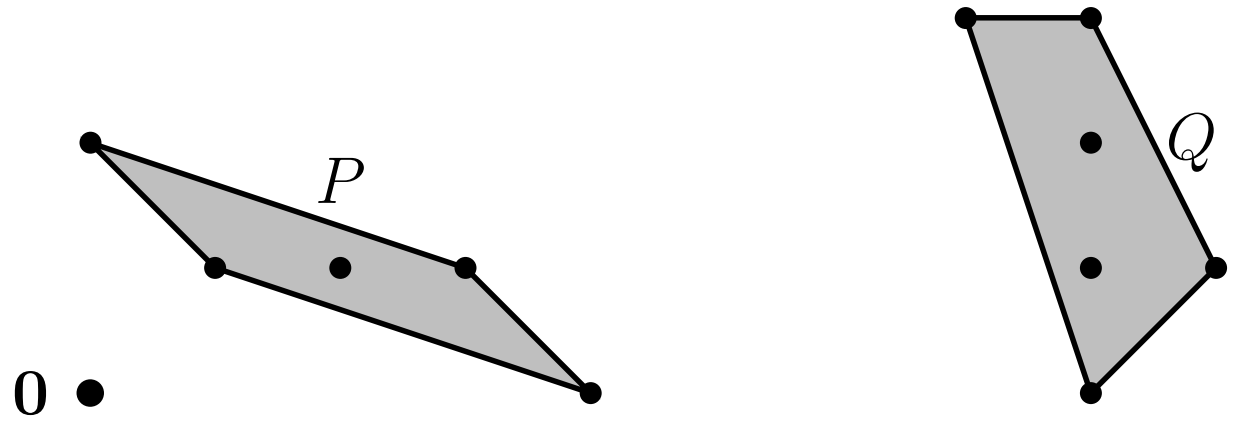
For **fixed-dim. polytopes** $P, Q \subseteq \mathbb{R}^d$, testing

$$\text{int.cone}(P \cap \mathbb{Z}^d) \cap Q \neq \emptyset$$

is doable in **poly-time** (actually $\text{inputlength}^{2^{O(d)}}$).



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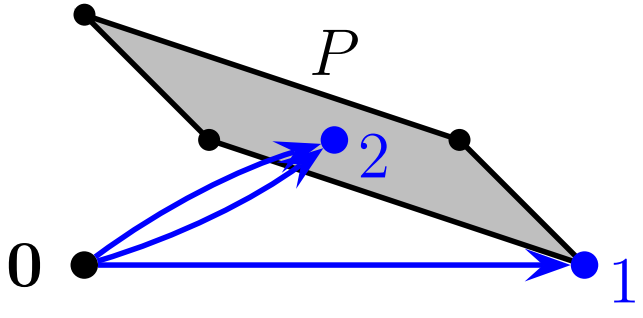

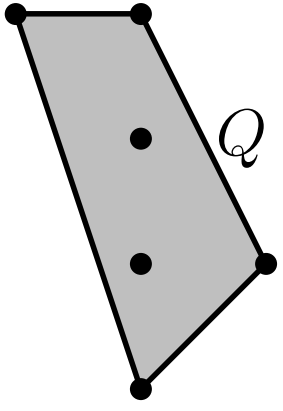

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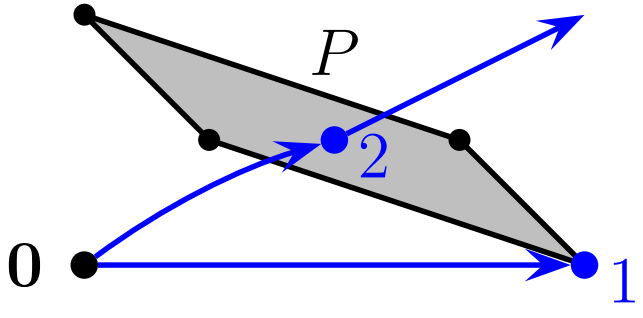

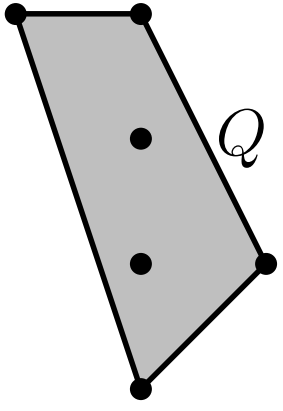

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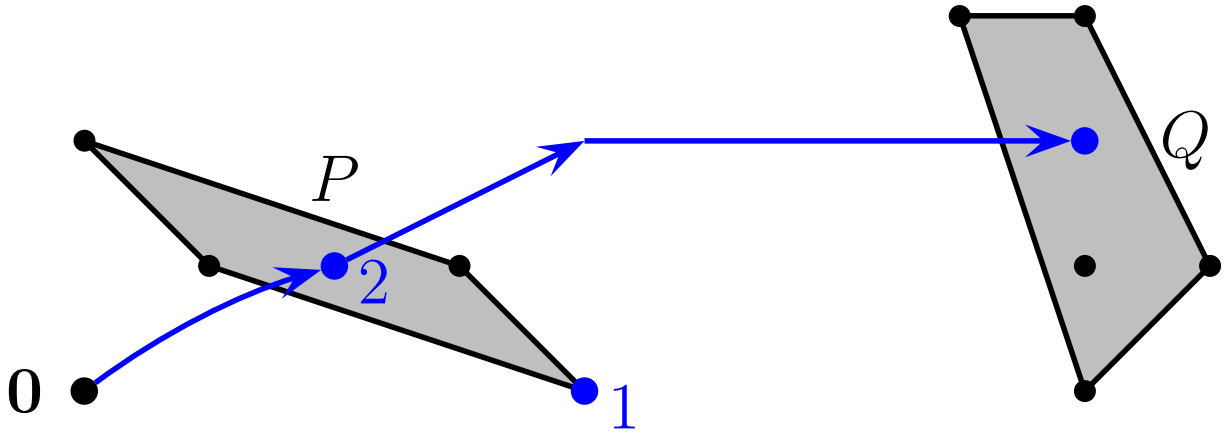

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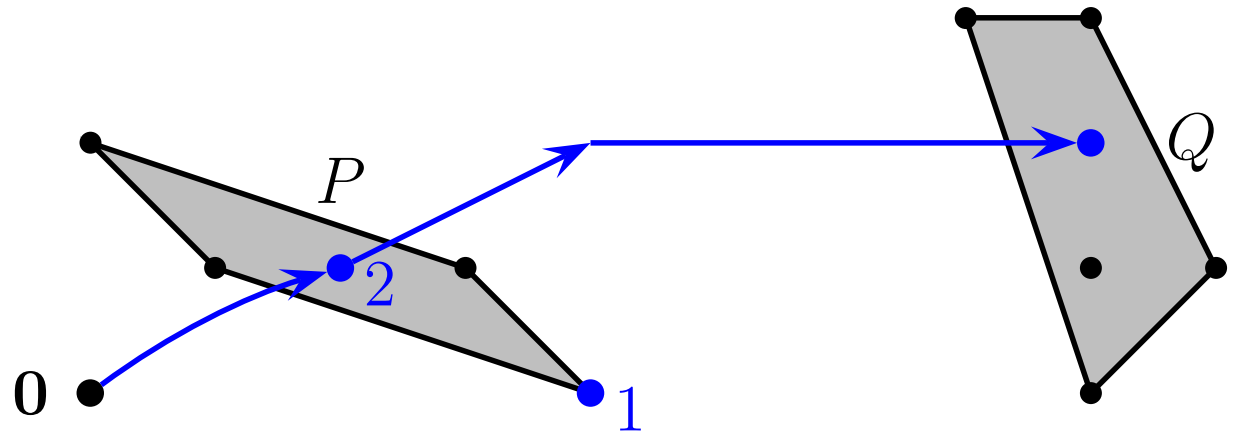

- For Bin Packing:
  $P := \{\binom{x}{1} \mid s^T x \leq 1,\ x \geq \mathbf{0}\}$ and $Q := \{\binom{a}{OPT}\}$

Int. conic combinations

Theorem (Eisenbrand & Shmonin '06)

If $P \subseteq \mathbb{R}^d$ convex, then any integer conic combination

$$a = \sum_{x \in P \cap \mathbb{Z}^d} \lambda_x \cdot x$$

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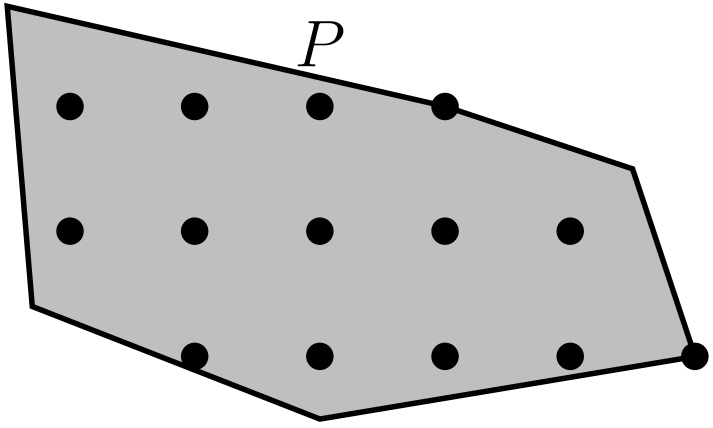

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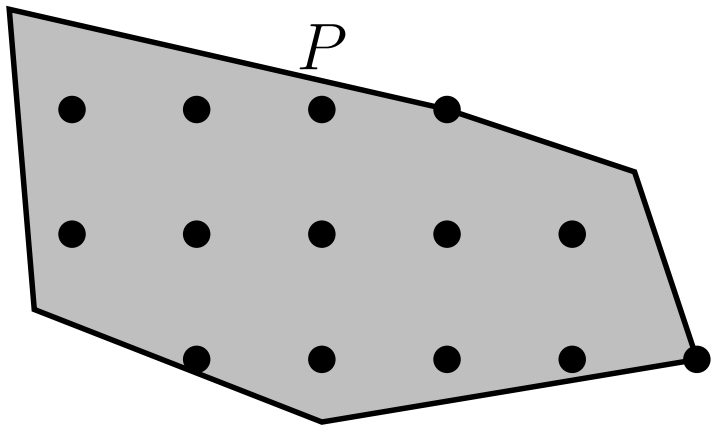

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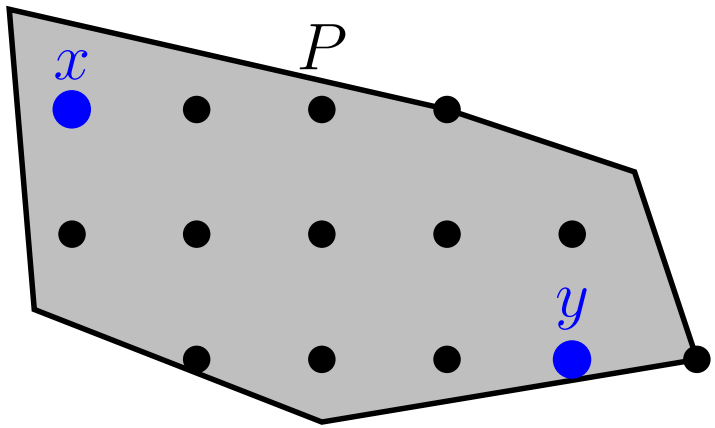

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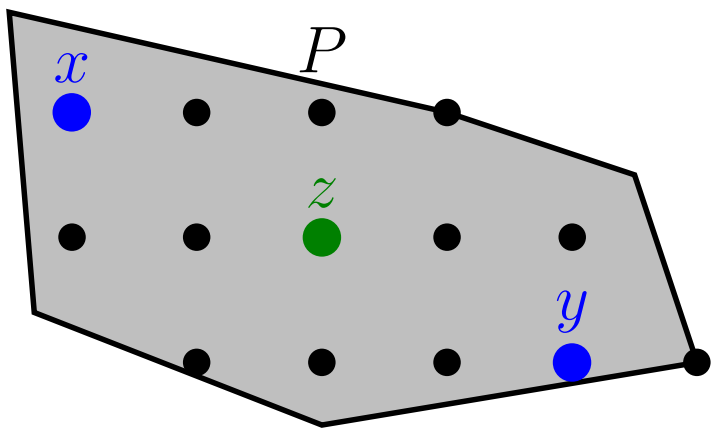

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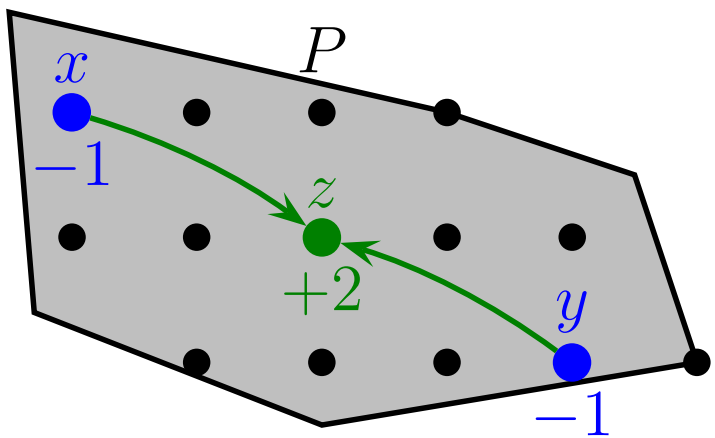

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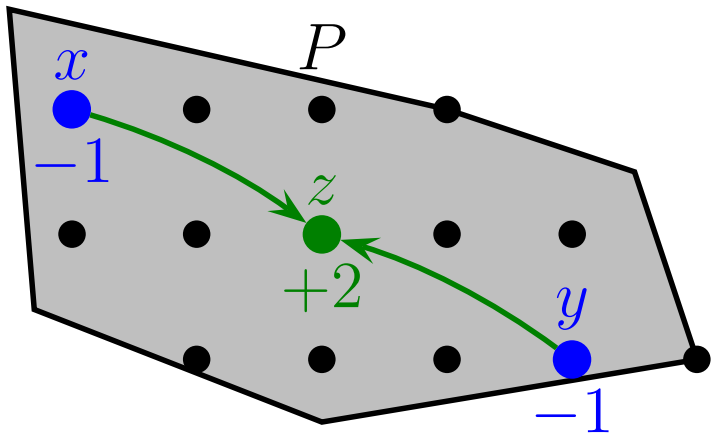

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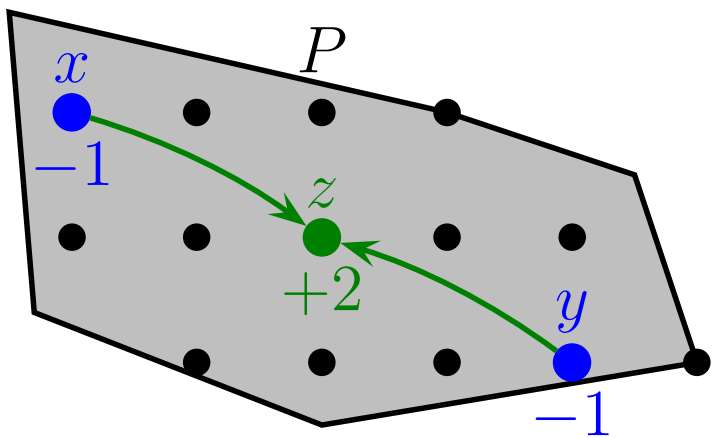
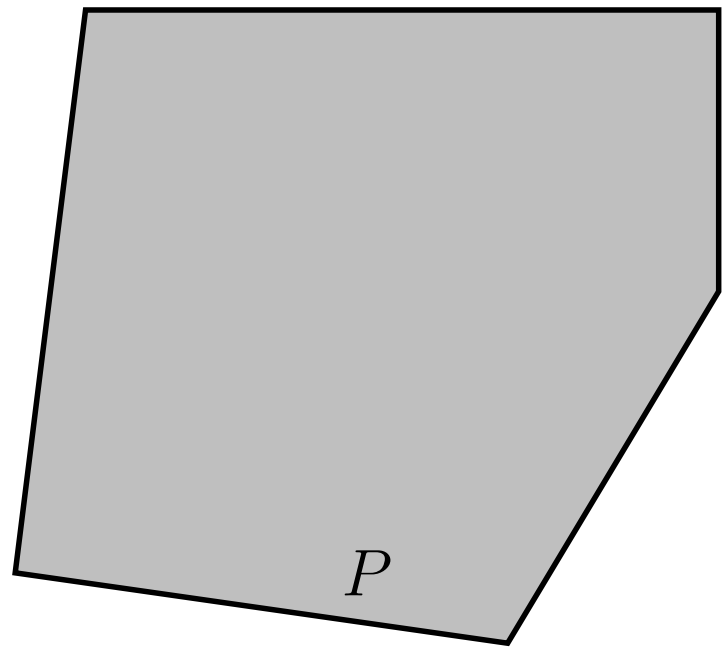


- **Problem:** Still don't know which points to take!

Redistributing weight



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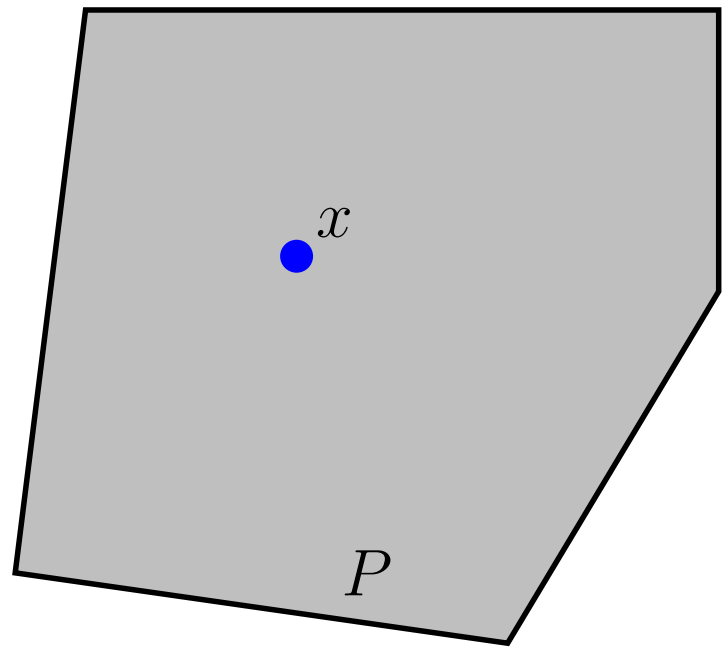
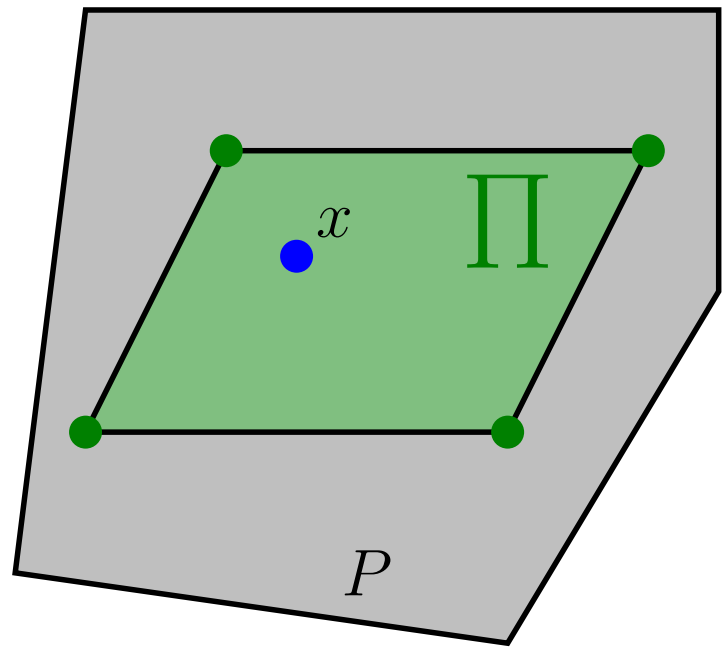


Redistributing weight

- Consider **parallelepiped** $\Pi \ni x$ with integral vertices



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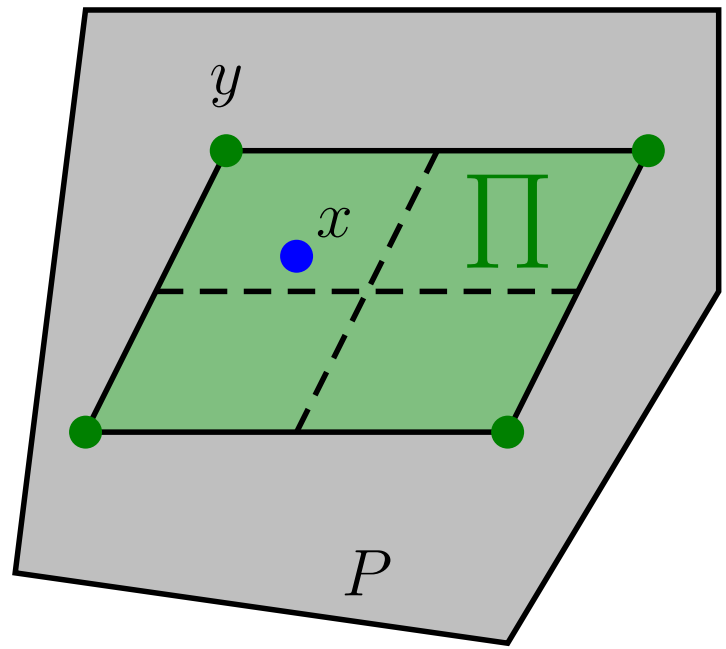

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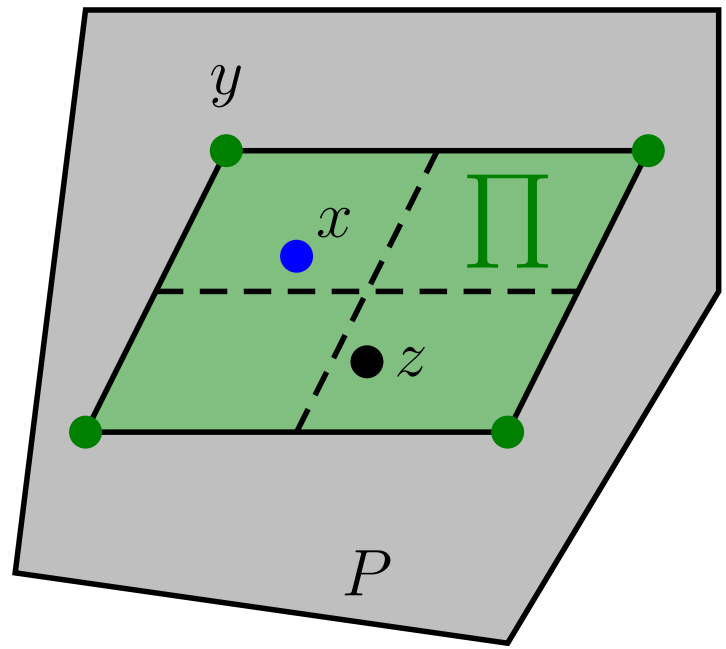

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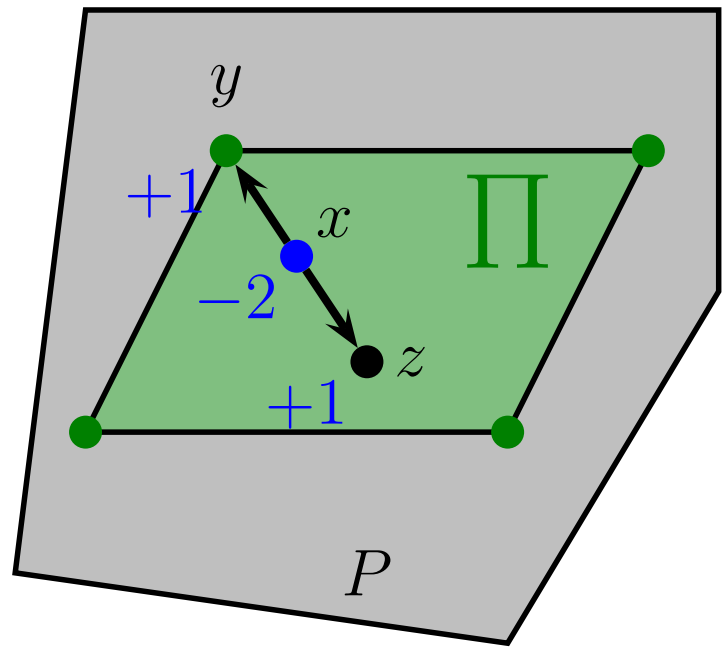

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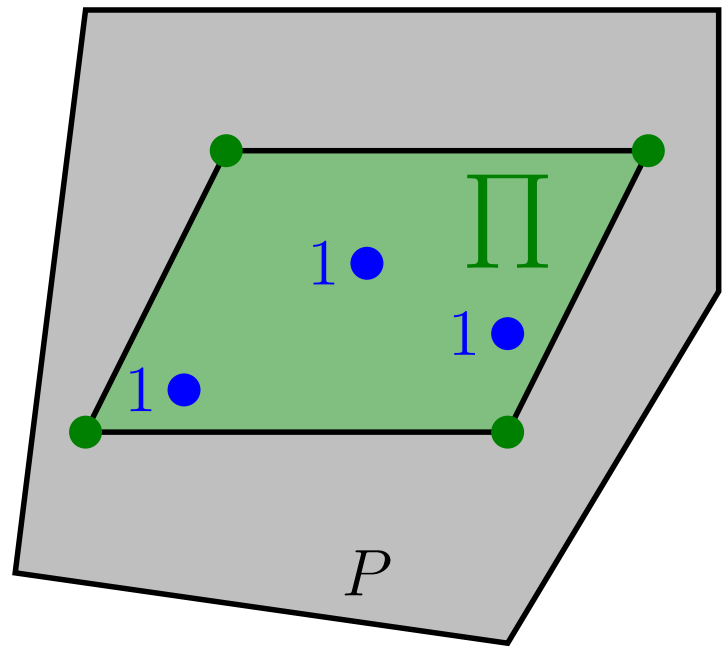

Redistributing weight

Lemma

For x in parallelepiped Π and $\lambda_x \in \mathbb{N}$, one can write

$$\lambda_x x = \text{int.cone(vertices of } \Pi) + \sum \text{ of } 2^d \text{ points in } \Pi \cap \mathbb{Z}^d$$

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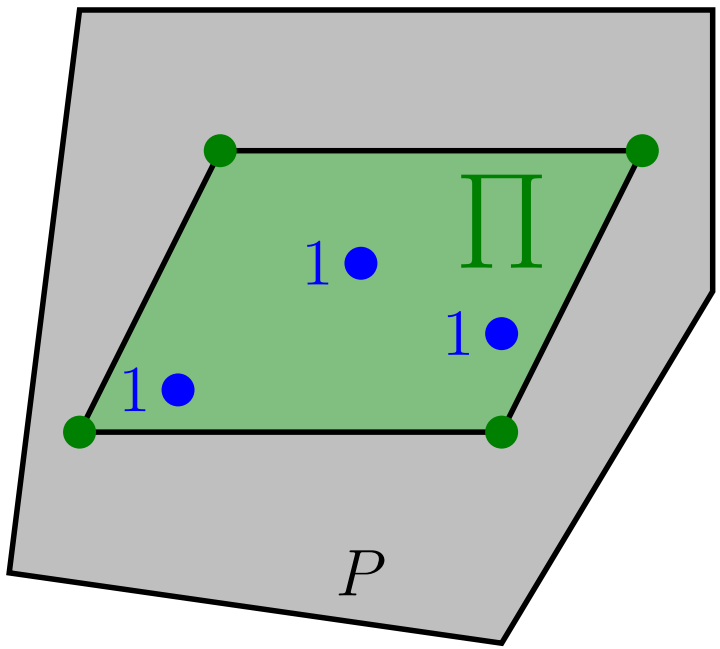
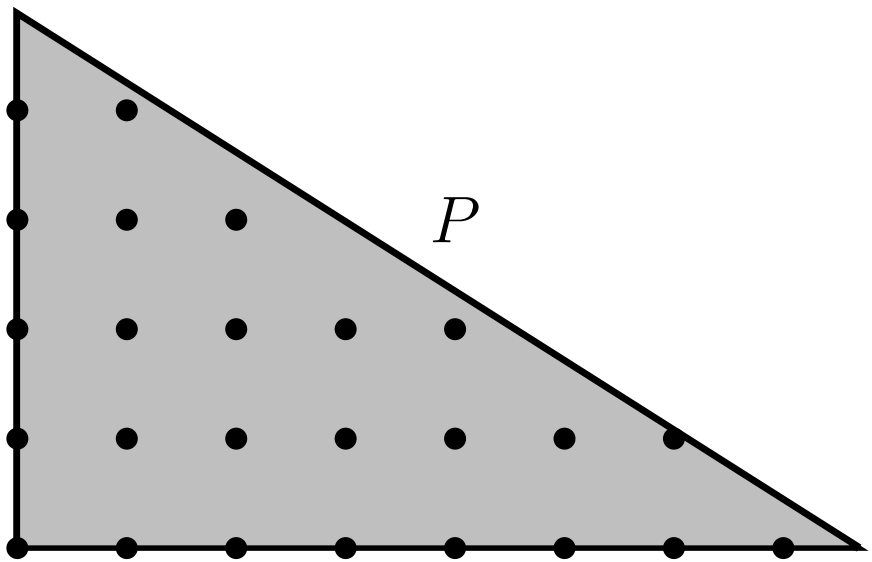


Covering a polytope with parallelepipeds

Lemma

For fixed-dim $P \subseteq \mathbb{R}^d$, we can cover $P \cap \mathbb{Z}^d$ with **poly-many parallelepipeds** (with int. vertices and $\subseteq P$).



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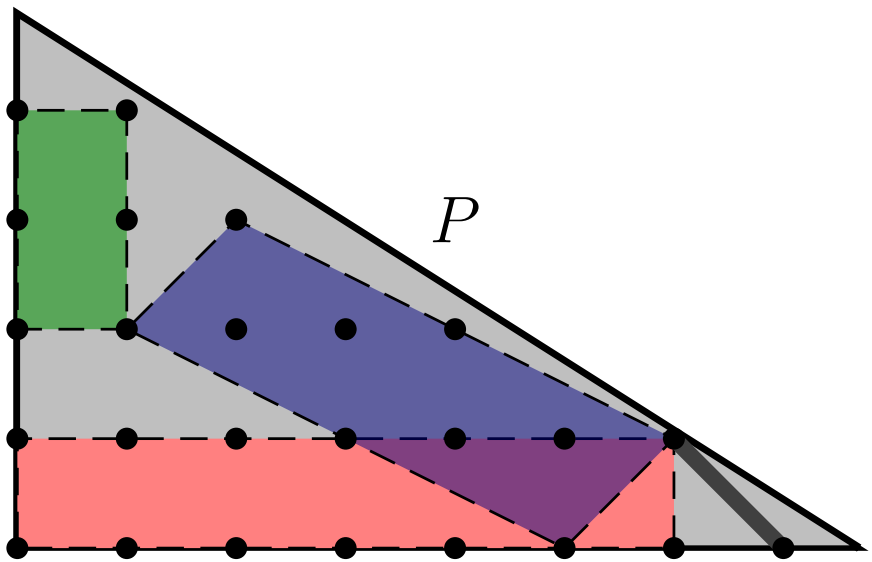
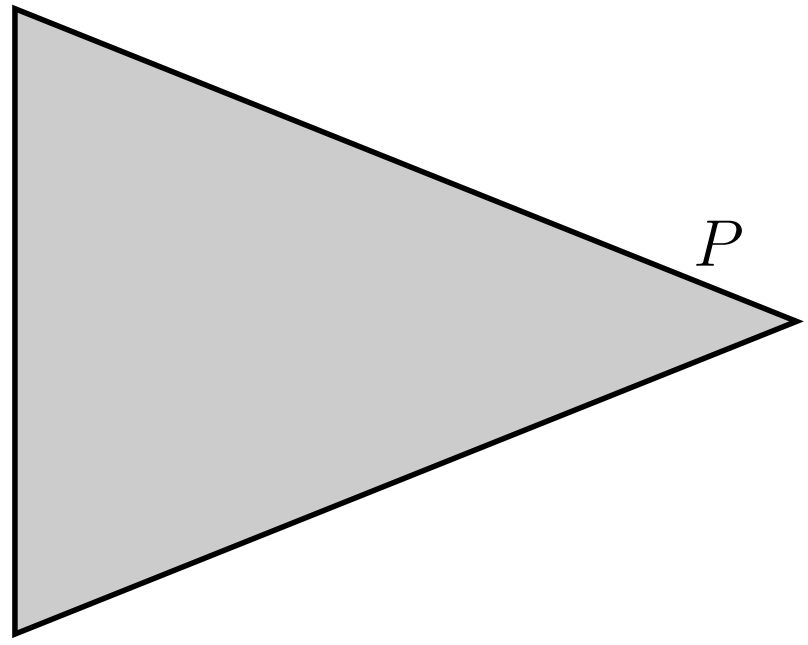


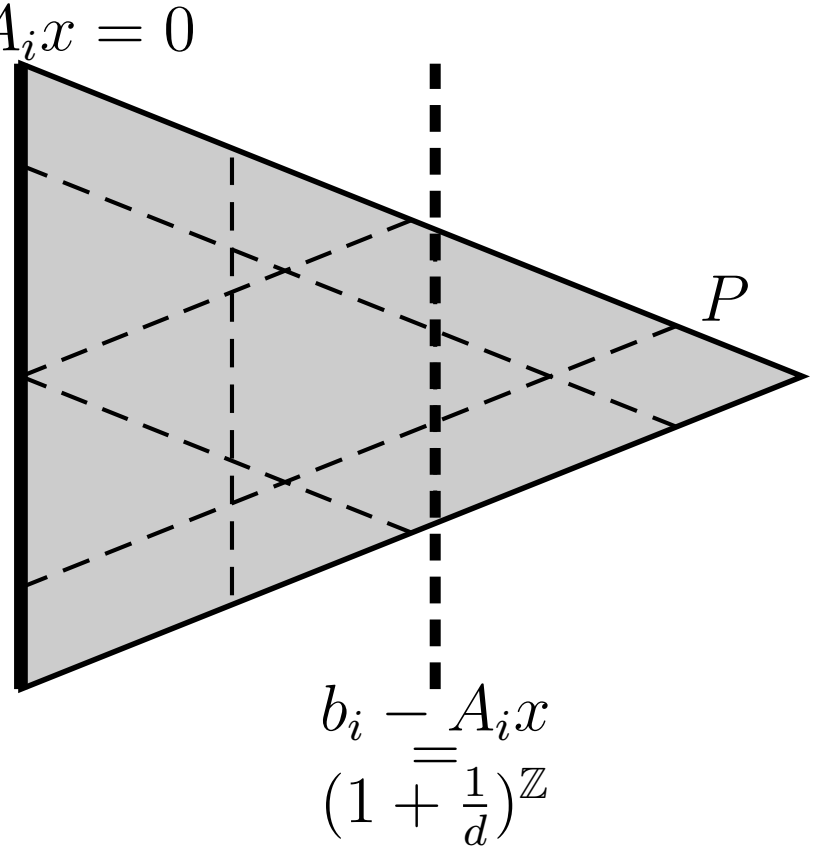
Covering a polytope w. parallelep. (2)



Covering a polytope w. parallelep. (2)

- Split $P = \{x \mid Ax \leq b\}$ into poly many **cells**

$C = \{x \mid \alpha_{j(i)} \leq A_i x \leq \alpha_{j(i)+1}\}$



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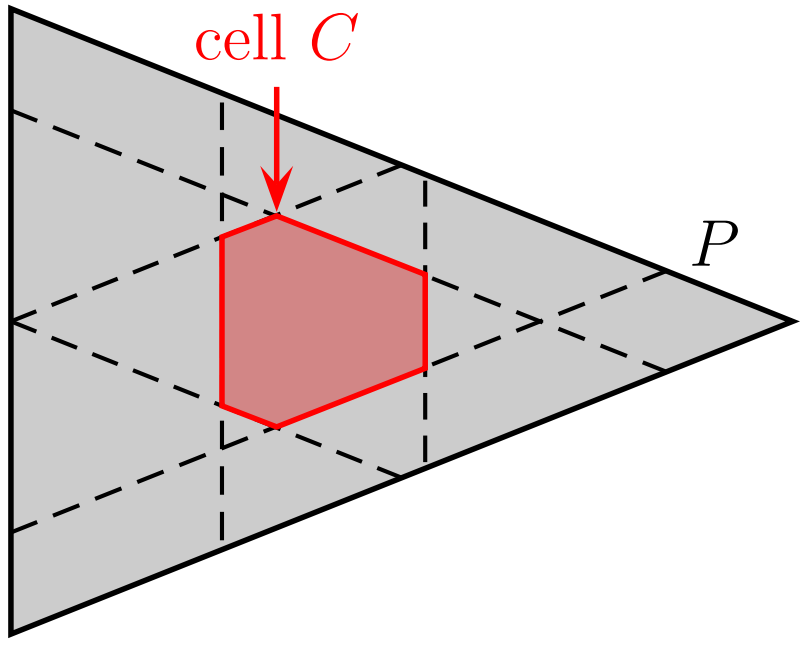

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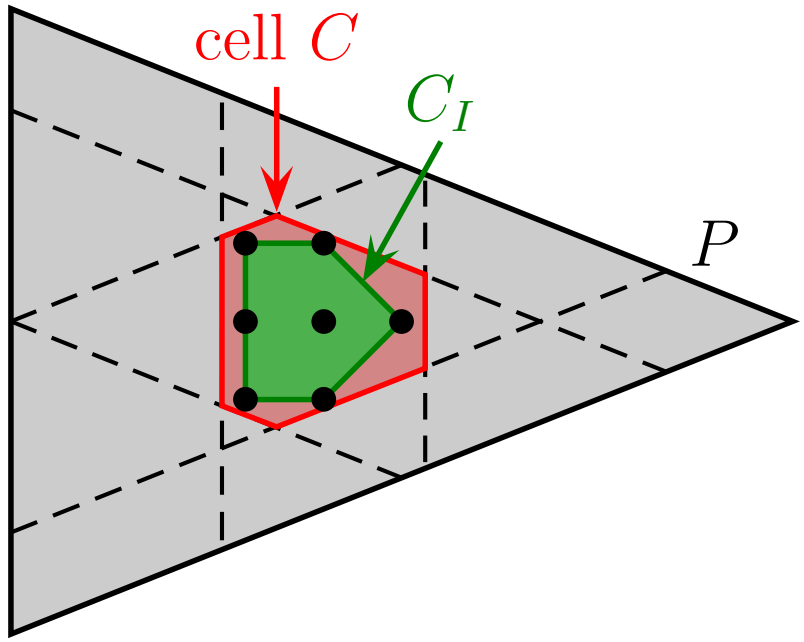

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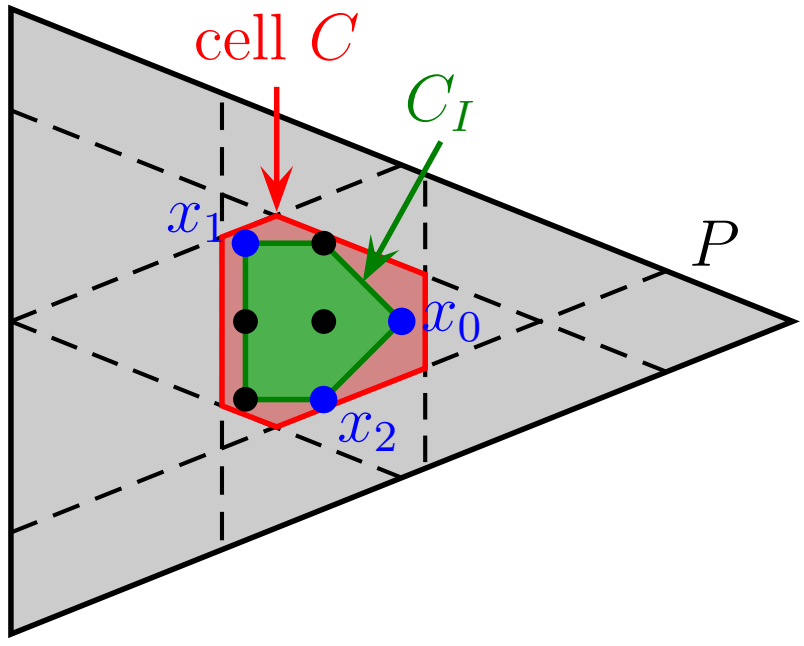

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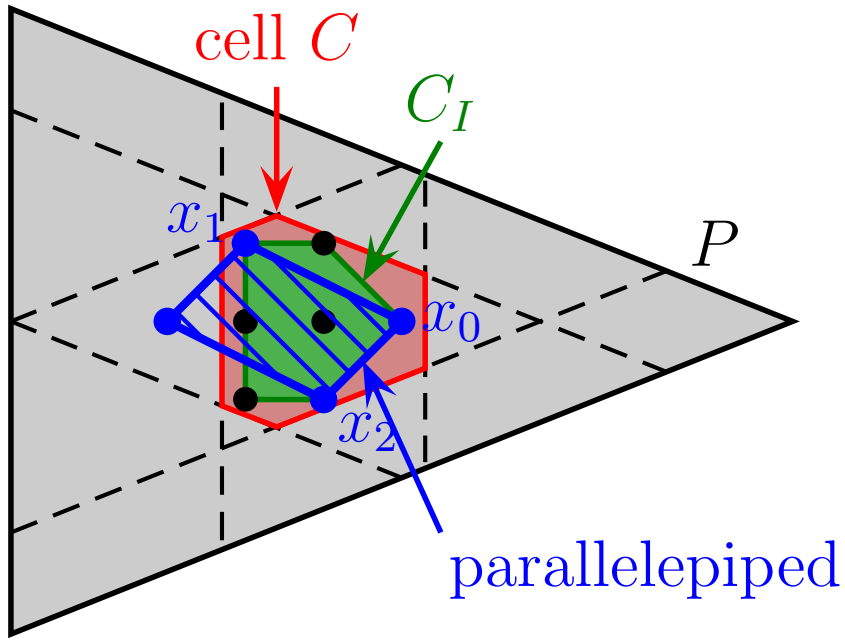

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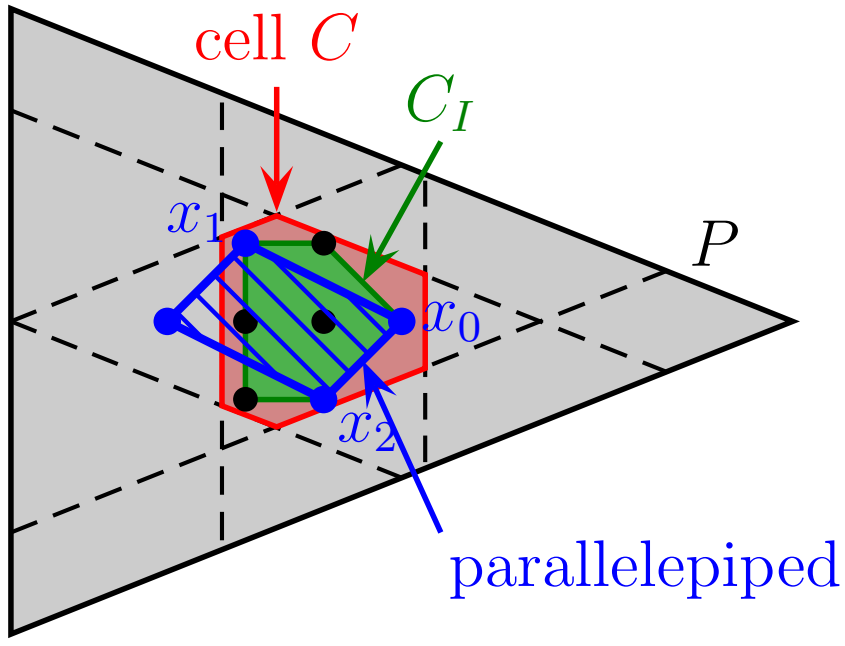

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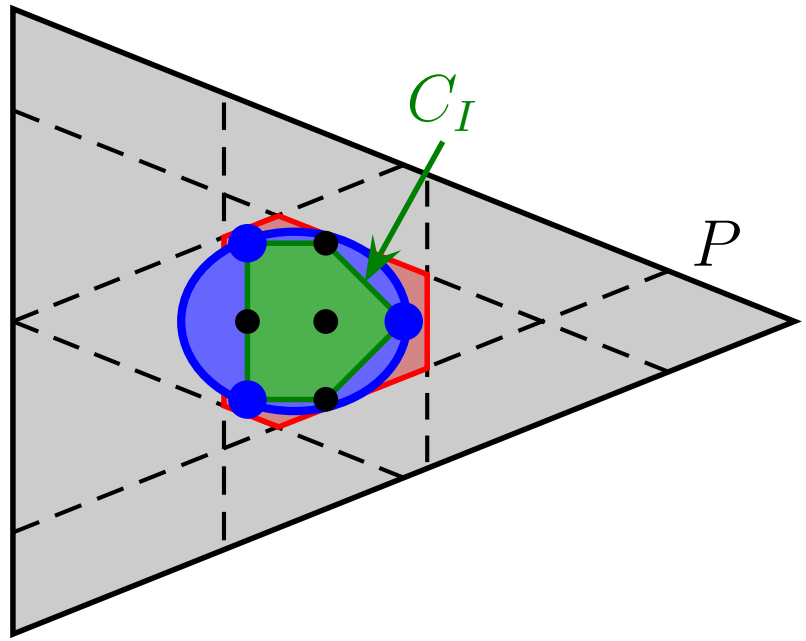

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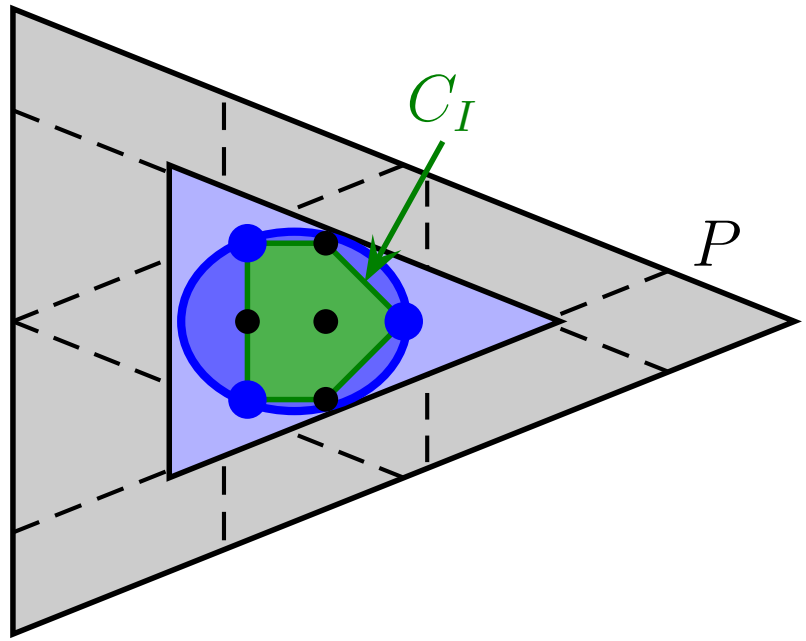

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- **Input:** polytopes P, Q in ineq. description
- **Output:** Coefficients for int.cone$(P \cap \mathbb{Z}^d) \cap Q \neq \emptyset$

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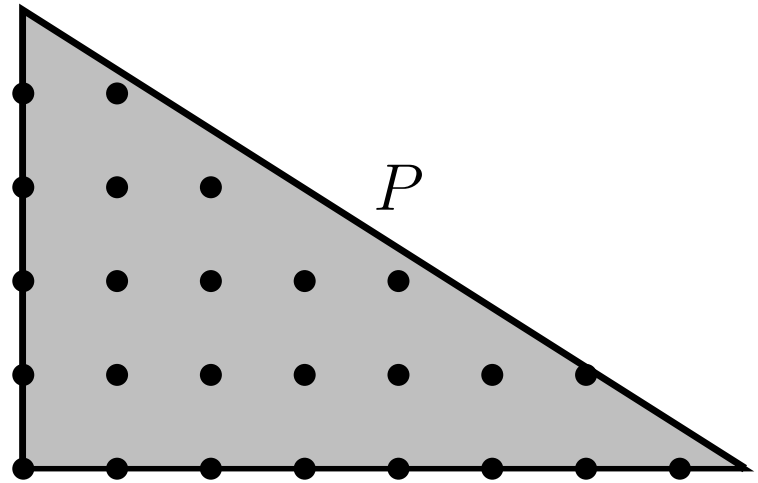

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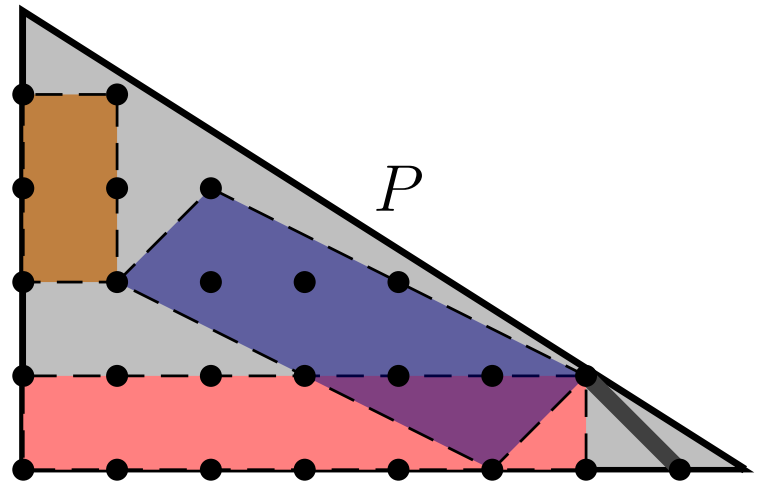

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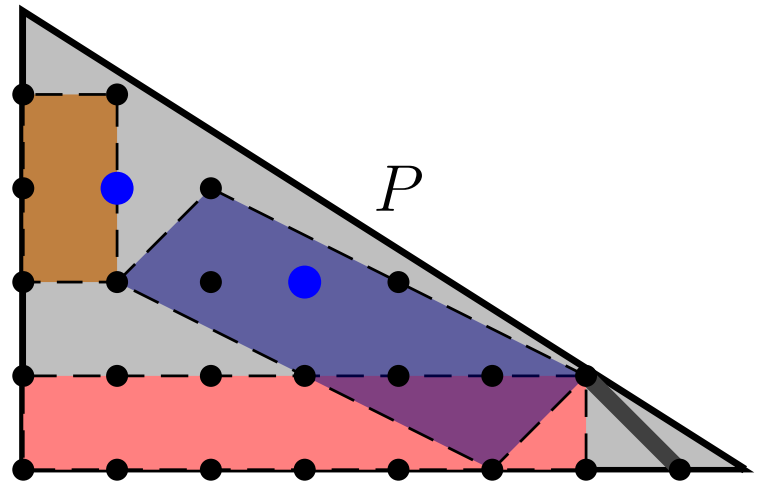

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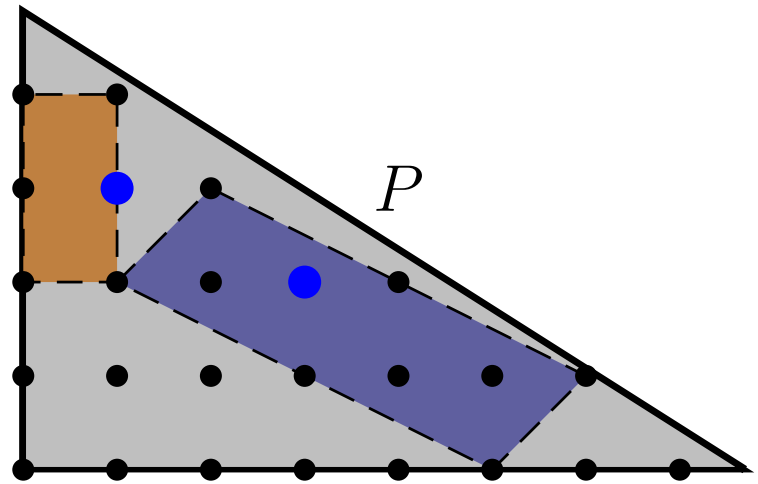

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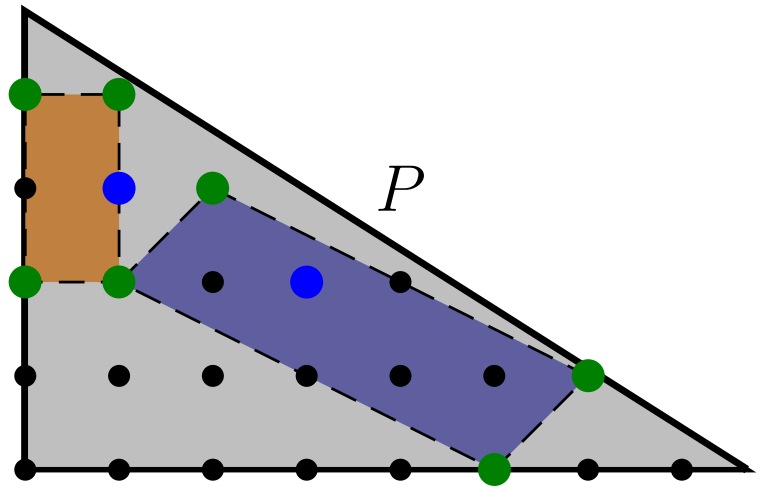

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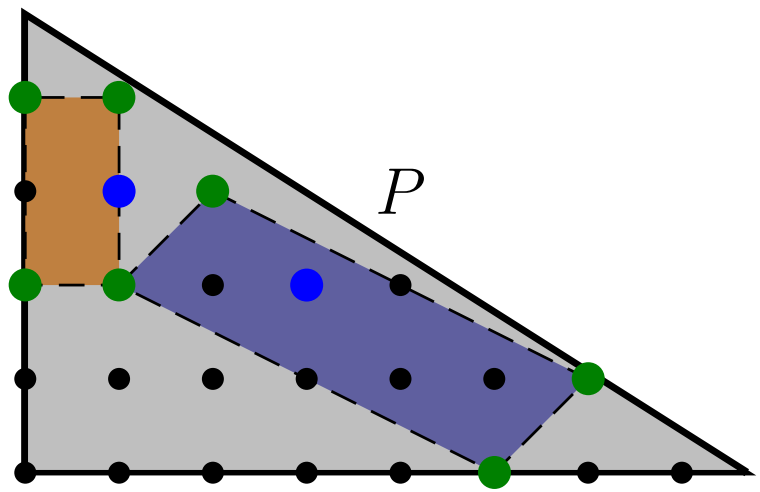

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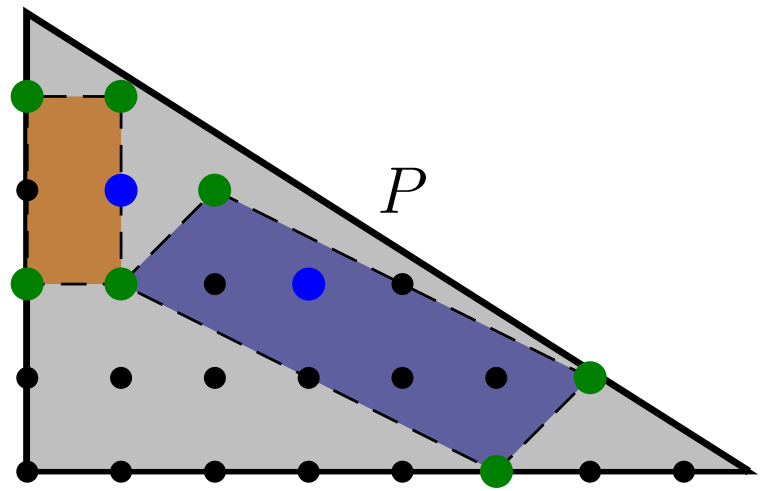
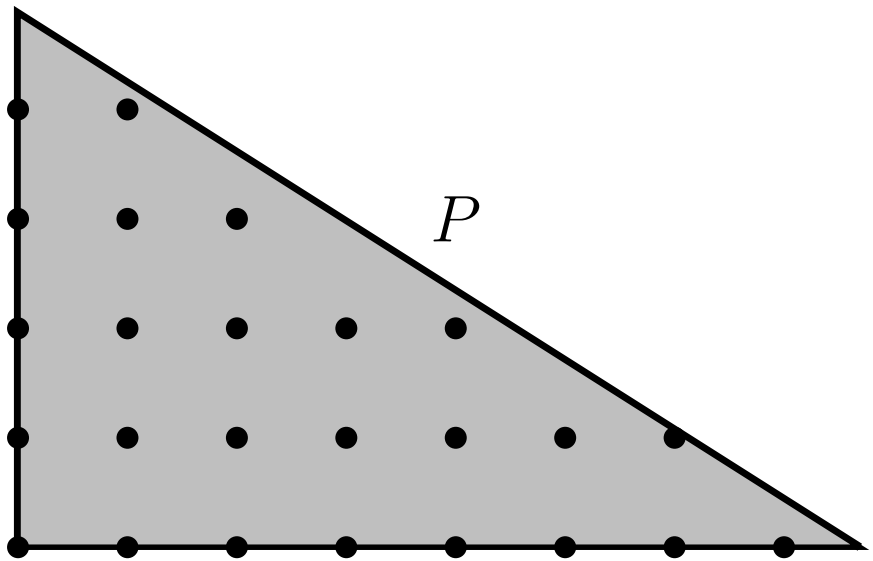


A Structure Theorem

Structure Theorem

For polytope P



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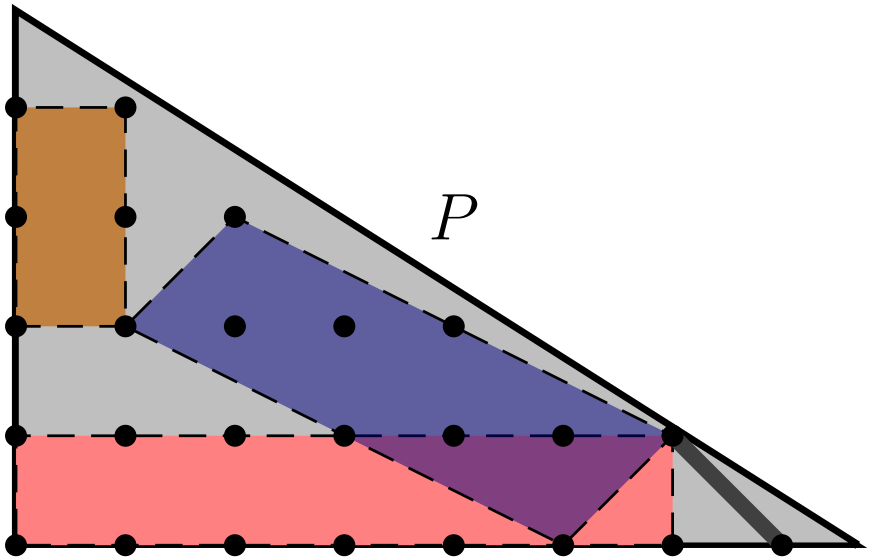

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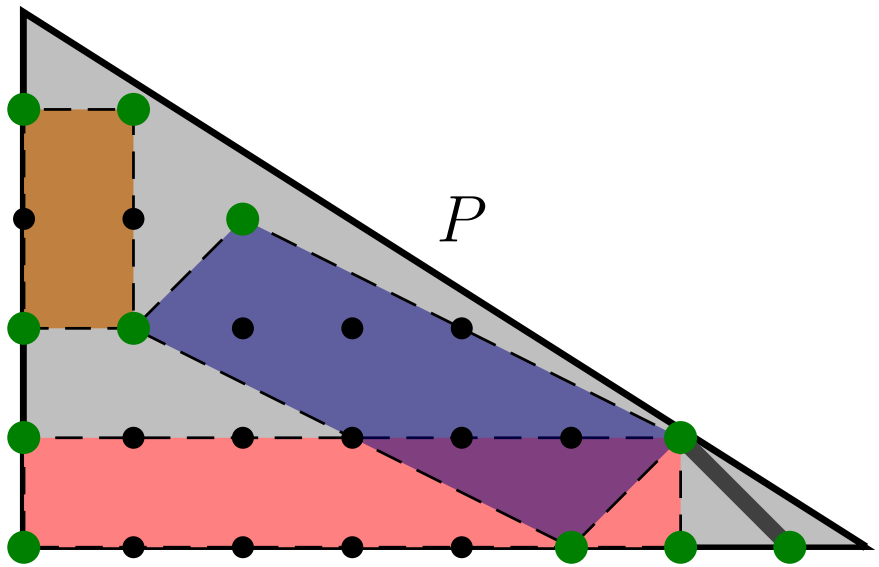

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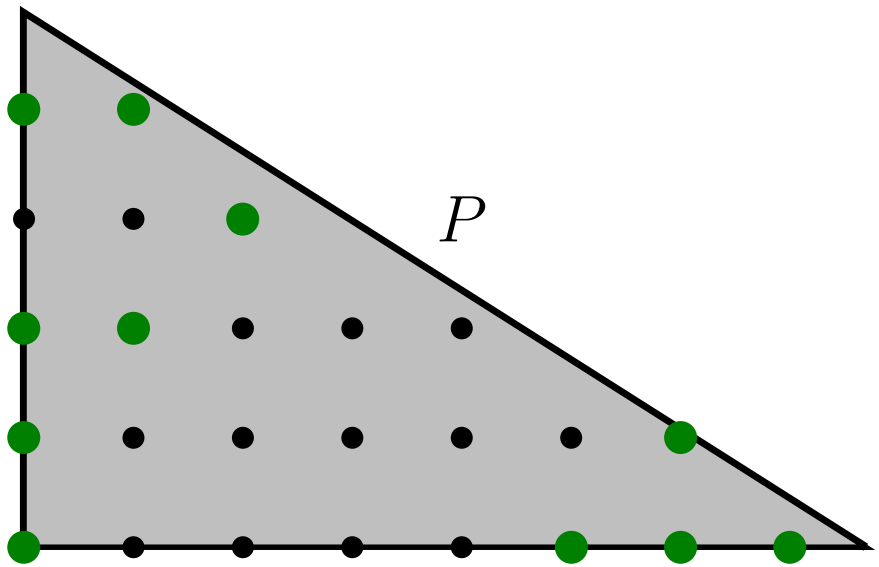

A Structure Theorem

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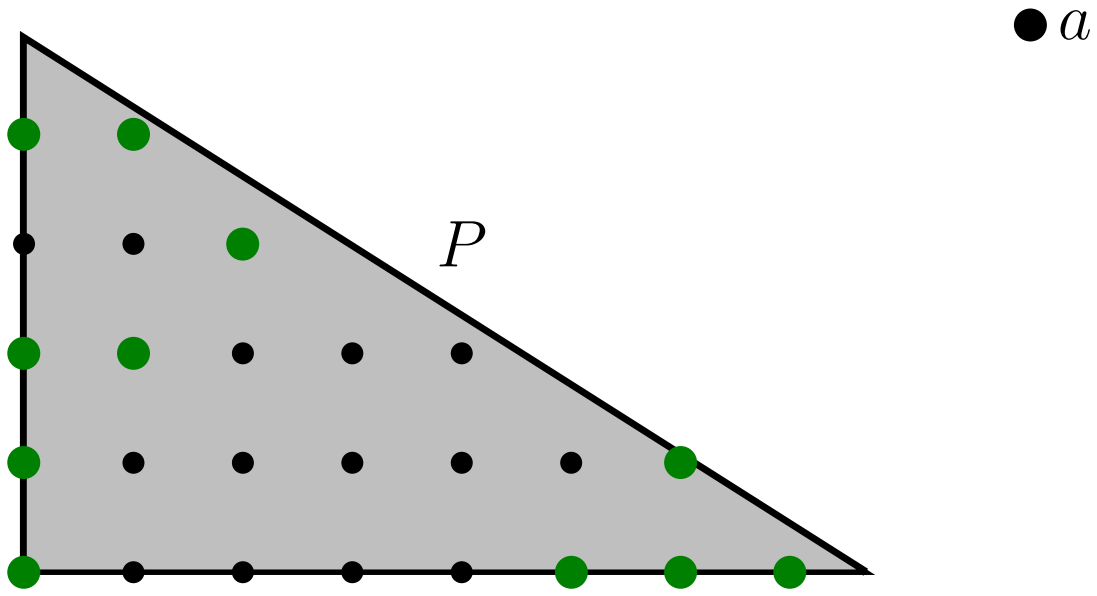
For polytope P, $\exists$ poly-time comp. set $\boldsymbol{X} \subseteq P \cap \mathbb{Z}^d$ s.t. **for all** $a \in$ **int.cone**$(P \cap \mathbb{Z}^d)$ one can express

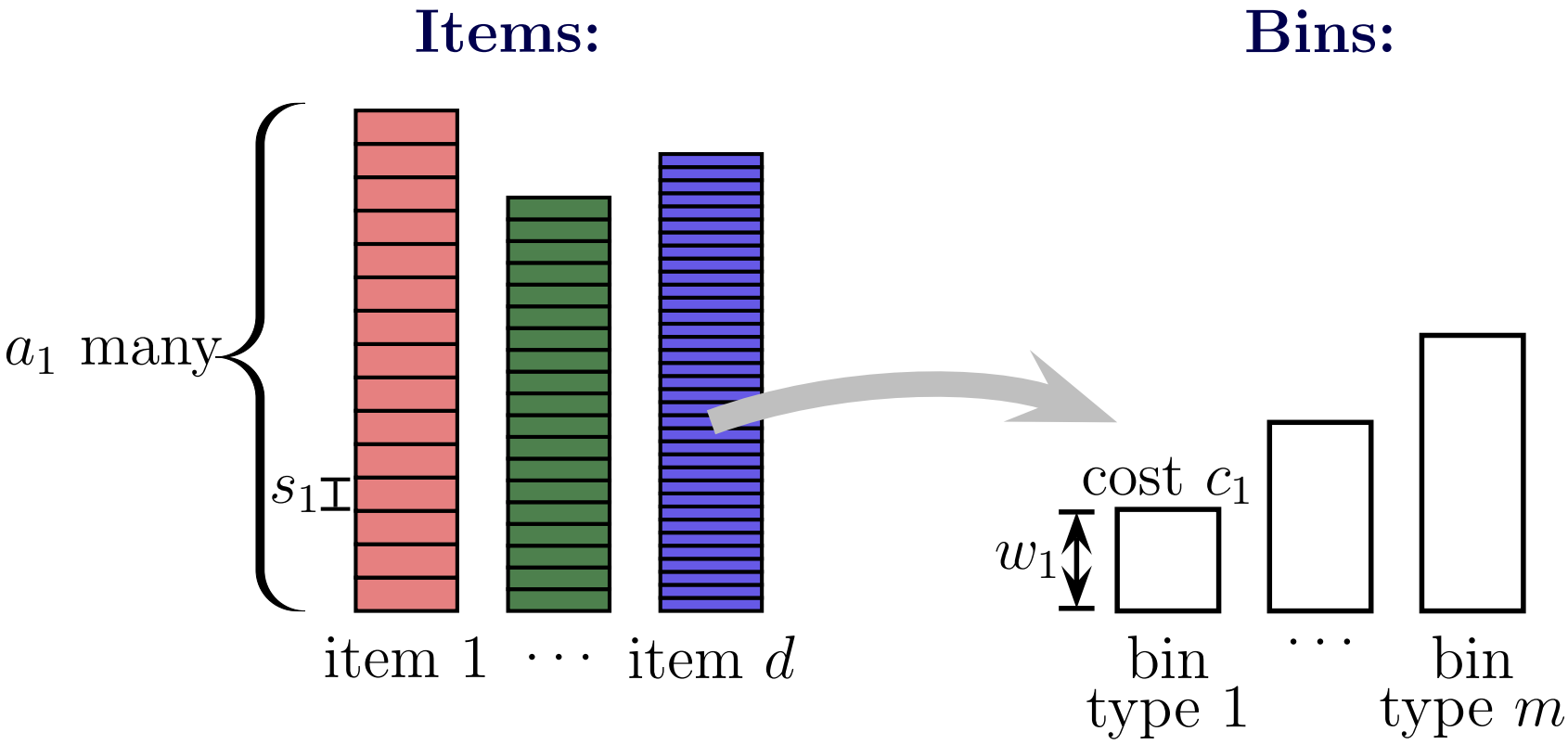
$$a = \text{int.cone}(2^{2d} \text{ points in } X) + \sum \text{of } 2^{2d} \text{ points in } P \cap \mathbb{Z}^d$$



More applications (1)

Theorem

Cutting stock with $d = O(1)$ item types and $m = O(1)$ of bin types can be solved in **polytime**.



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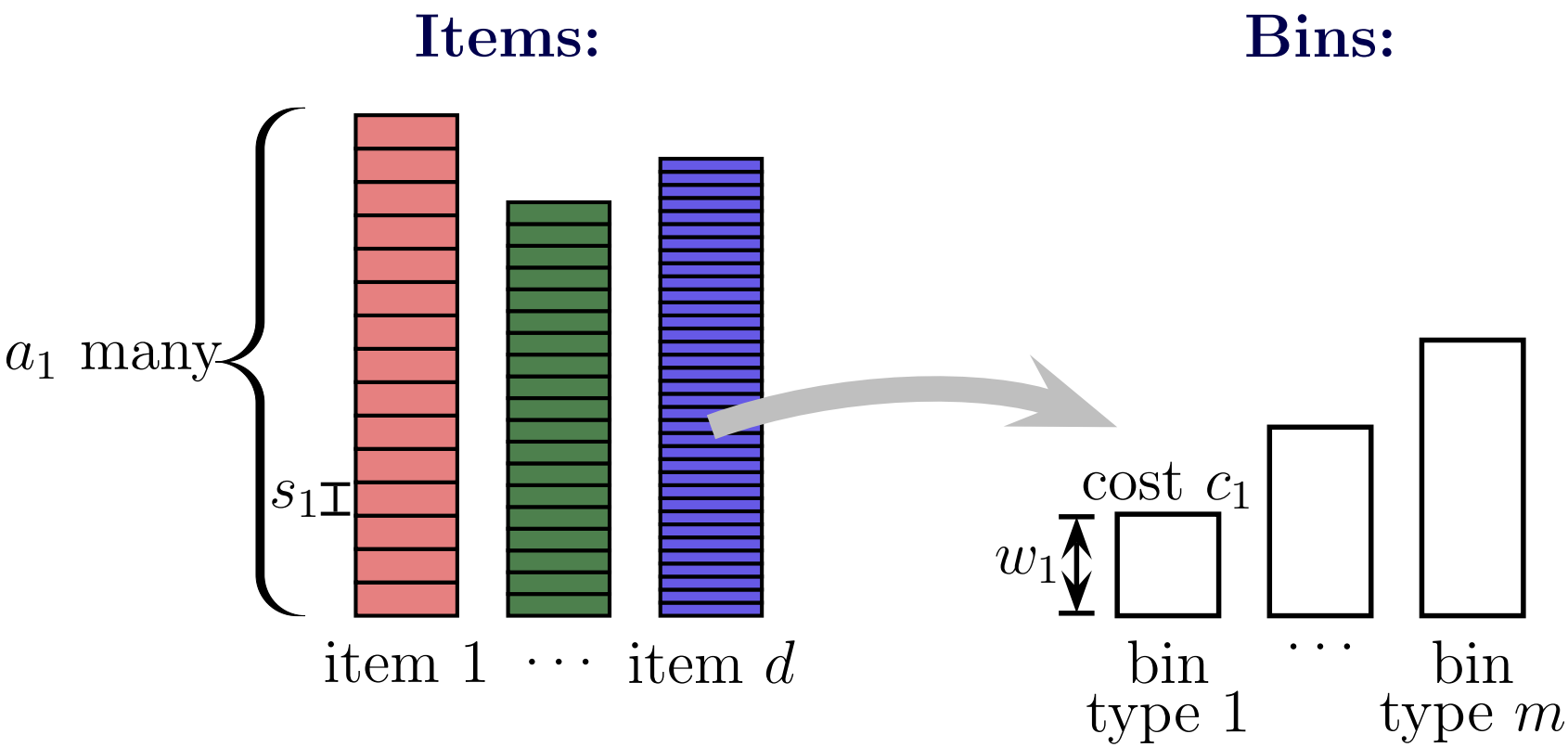

- $P = P_1 \times \ldots \times P_m$ with $P_j := \{ \binom{x}{c_j} \mid s^T x \leq w_j \}$

More applications (2)

High Multiplicity Scheduling:

- **Input:** d job types, each with release time, deadline, processing time, multiplicity
- **Goal:** Schedule jobs to minimize number of used machines

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The end

Thanks for your attention