Lecturer: Thomas Rothvoss Due date: Friday, Feb 17, 2023, 11pm, on GradeScope

Problem Set 6

CSE 599S - Lattices

Winter 2023

Exercise 2.4 (10pts)
Let $\boldsymbol{B} \in \mathbb{R}^{n \times n}$ be a regular matrix. Prove that $\mathcal{V} \subseteq n \cdot \lambda_n(\Lambda) \cdot B_2^n$ where $\mathcal{V}$ is the Voronoi cell of the lattice $\Lambda := \Lambda(\boldsymbol{B})$.

Exercise 2.7 (10pts)
Prove the following statement: Let $\Lambda \subseteq \mathbb{R}^n$ be a full rank lattice and let $\boldsymbol{t} \in \mathbb{R}^n$ be a vector with $\textsc{Cvp}(\Lambda, \boldsymbol{t}) < 2^{-n/2-1}\lambda_1(\Lambda)$. Then one can find a vector $\boldsymbol{x} \in \Lambda$ with $\|\boldsymbol{x} - \boldsymbol{t}\|_2 = \textsc{Cvp}(\Lambda, \boldsymbol{t})$ in polynomial time.

Exercise 3.3 (10pts)
Let $\Lambda \subseteq \mathbb{R}^n$ be a full rank lattice and let $K \subseteq \mathbb{R}^n$ be a symmetric convex body. Then for any $t > 0$, $|\Lambda \cap t \cdot \lambda_1(\Lambda, K) \cdot K| \leq (2t+1)^n$.