Lecturer: Thomas Rothvoss | Due date: Friday, Oct 10, 2025, 11pm, on GradeScope

Problem Set 2

Math 581A - Analysis of Boolean Functions

Fall 2025

Exercise 2.1 (10 points)
In the following, let $f : \{-1,1\}^n \to \{-1,1\}$ be a boolean function.

(i) Prove that $|\hat{f}(\{i\})| \leq \mathrm{Inf}_i[f]$ for all $i \in [n]$.
Hint. It might be useful to have the following fact: For a random variable $X \in \{-1,0,1\}$ one has $|\mathbb{E}[X]| \leq \Pr[X \neq 0]$.

(ii) Prove that

$$\sum_{i=1}^{n} |\hat{f}(\{i\})| \leq \deg(f)$$

Remark. It is an open problem if the right hand side can be improved to $O(\sqrt{\deg(f)})$.

Exercise 2.2 (10 points)
Solve the following:

(i) Prove that for any function $f : \{\pm 1\}^n \to \mathbb{R}$ and any $i \in [n]$ one has $\hat{f}(\{i\}) = \mathbb{E}[D_i f(x)]$.

A function $f : \{\pm 1\}^n \to \{-1,1\}$ is called *monotone* if for all $x, y \in \{-1,1\}^n$ one has $x \leq y \Rightarrow f(x) \leq f(y)$.

(ii) Prove that any monotone function $f : \{\pm 1\}^n \to \{-1,1\}$ has $D_i f(x) \in \{0,1\}$ for all $x \in \{-1,1\}^n$ and all $i \in [n]$.

(iii) Prove that for any monotone function $f : \{\pm 1\}^n \to \{-1,1\}$ one has $\mathrm{Inf}_i[f] = \hat{f}(\{i\})$ for all $i \in [n]$.
Hint. Use (i)+(ii).