Lecturer: Thomas Rothvoss Due date: Friday, Oct 3, 2025, 11pm, on GradeScope

Problem Set 1

Math 581A - Analysis of Boolean Functions

Fall 2025

Exercise 1.1[1] **(10pts)**
Let $f : \{-1,1\}^n \to \{0,1\}$ and assume that $|\{x \in \{-1,1\}^n : f(x) = 1\}|$ is odd. Prove that all of f's Fourier coefficients are non-zero.

Exercise 1.2[2] **(10pts)**
Consider a function $f : \{-1,1\}^n \to \mathbb{R}$. We define its *extension* $F : \mathbb{R}^n \to \mathbb{R}$ as the function with $F(x) := \sum_{S \subseteq [n]} \hat{f}(S) \cdot \prod_{i \in S} x_i$ for $x \in \mathbb{R}^n$. Show that for any $\mu \in [-1,1]^n$ one has

$$F(\mu) = \mathop{\mathbb{E}}_{y \sim \mu}[f(y)]$$

where $y \sim \mu$ produces a random vector $y \in \{-1,1\}^n$ so that $\mathbb{E}[y_i] = \mu_i$ for all $i = 1,\dots,n$ and the coordinates of y are independent.

[1]Problem is from O'Donnell's textbook where it is problem 1.3
[2]Problem is from O'Donnell's textbook where it is problem 1.4