Problem Set 5

CSE 531 - Computational Complexity

Winter 2024

Exercise 4.12 (from the book of Arora and Barak; 10pts)
Define $\text{polyL} := \bigcup_{c > 0} \text{DSpace}((\log n)^c)$. Steve’s Class $\text{SC}$ (named after Steve Cook) is defined to be the set of languages $L \subseteq \{0,1\}^*$ that can be decided by a deterministic Turing machine that runs in polynomial time and $(\log n)^c$ space, for some constant $c > 0$. It is an open problem whether $\text{PATH} \in \text{SC}$. Why does Savitch’s Theorem not resolve this question? Is $\text{SC}$ the same as $\text{polyL} \cap \text{P}$?

Remark. This question should be understood as being somewhat open ended.

Exercise 2 (10pts)
Consider the language $BIPARTITE = \left\{ G \mid G = (V,E) \text{ is undirected graph and } \exists \text{ partition } V = V_0 \cup V_1 : \text{all edges in } E \text{ run between } V_0 \text{ and } V_1 \right\}$.

Prove that $BIPARTITE \in \text{NL}$.

Hint. First prove that $G \in BIPARTITE$ if and only if $G$ does not contain an odd length cycle. Next, make use of a theorem we have proven in class.