Lecturer: Thomas Rothvoss Due date: Friday, Jan 12, 2024, 11pm, on GradeScope

Problem Set 1

CSE 531 - Computational Complexity

Winter 2024

Exercise 1.9 (slightly modified; 10pts)

Define *RAM Turing machine* to be a Turing machine M that has *random access memory*. We formalize this as follows: The machine has an infinite array A that is initialized to be all blanks. It accesses the array as follows. One of the machine's work tapes is designated as *address tape*. Also the machine has two special alphabet symbols denoted by R and W and an additional state we denote by q_{access}. Whenever the machine enters q_{access}, if its adress tape contains $\lfloor i \rfloor \mathsf{R}$ (where $\lfloor i \rfloor$ denotes the binary representation of $i \in \mathbb{Z}_{\geq 0}$), then the value $A[i]$ is written in the cell next to the R. If its tape contains $\lfloor i \rfloor \mathsf{W} \sigma$ (where σ is some symbol in the machine's alphabet), then $A[i]$ is set to the value σ.

Show that if a function $f : \{0,1\}^* \to \{0,1\}^*$ is computable within time $T(n)$ (for some time constructable T) by a RAM TM, then it is computable in time $O(T(n)^3)$ by a "standard" TM $\tilde{M}$.

Bonus. The book by Arora & Barak claims a running time of $O(T(n)^2)$ is possible but I am not quite convinced. One bonus point if you manage!