Lecturer: Thomas Rothvoss Due date: Friday, Oct 4, 2024, 11pm, on GradeScope

Problem Set 1

CSE 521 - Design and Analysis of Algorithms

Fall 2024

Exercise 1 (10pts)
Given a graph $G=(V,E)$ with $n=|V|$ vertices and a parameter $\beta \in \mathbb{N}$, a cut is a β-*approximate min cut*, if the number of its edges is at most β times the minimum cut of G. Modify Karger's contraction algorithm so that it implies that any graph G has at most $n^{2\beta}$ many β-approximate min cuts. You would also receive full credit if you show that the number of β-approximate min cuts is at most $n^{O(\beta)}$.

Exercise 2 (10pts)
Consider adapting the min-cut algorithm to the problem of finding an s-t min-cut in an undirected graph. In this problem, we are given an undirected graph $G=(V,E)$ together with two distinguished vertices s and t. An s-t cut is a set of edges whose removal from G disconnects s from t; we seek an s-t cut of minimum cardinality. As the algorithm proceeds, the vertex s may get amalgamated into a new super-node as a result of an edge being contracted; we call this vertex the s-vertex (initially the s-vertex is s itself). Similarly, we have a t-vertex. As we run the contraction algorithm, we ensure that we never contract an edge between the s-vertex and the t-vertex, i.e., among all edges which are not between the s-vertex and the t-vertex we choose one uniformly at random. Show that there are graphs in which the probability that this algorithm finds an s-t min-cut is very small! How small this probability can be as a function of $n=|V|$, the number of vertices of G?