Lecturer: Thomas Rothvoss Due date: Friday, Nov 3, 2023, 11pm, on GradeScope

Problem Set 5

514 - Networks and Combinatorial Optimization

Autumn 2023

Exercise 5.2 (10pts)
Let $D = (V, A)$ be a directed graph. Prove that the node-edge incidence matrix $A \in \{-1, 0, 1\}^{V \times A}$ of D is totally unimodular.

Exercise 5.3 (10pts)
A set family $\mathcal{F} \subseteq 2^X$ is called *laminar* if for all $S, T \in \mathcal{F}$ one either has $S \subseteq T$ or $T \subseteq S$ or $S \cap T = \emptyset$. Let $A \in \{0, 1\}^{\mathcal{F} \times X}$ be a matrix whose rows are the characteristic vectors of $\mathcal{F}$. Prove that A is totally unimodular.
Example. The following is one such a matrix:

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$