

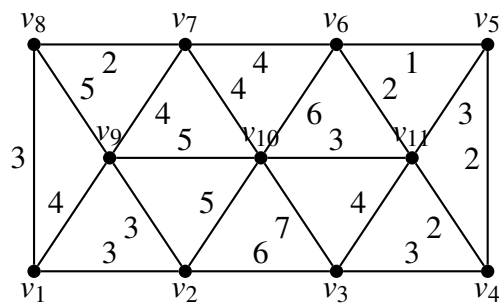
Problem Set 1

**514 - Networks and Combinatorial Optimization**

Autumn 2023

**Exercise 1.1 (10 pts)**

Find, both with the Dijkstra-Prim algorithm and with Kruskal’s algorithm, a spanning tree of minimum length in the graph in the figure below (it suffices to give a possible sequence of edges that are selected in both cases plus the length of the computed MST).



**Exercise 1.2 (10 pts)**

Let  $G = (V, E)$  be a graph and let  $\ell : E \rightarrow \mathbb{R}$  be a length function. Call a forest  $F$  *good* if  $\ell(F') \geq \ell(F)$  for each forest satisfying  $|F'| = |F|$ .

Let  $F$  be a good forest and  $e$  be an edge not in  $F$  so that  $F \cup \{e\}$  is a forest and such that (among all such  $e$ )  $\ell(e)$  is as small as possible. Show that  $F \cup \{e\}$  is good again.

**Remark.** Both exercises are taken verbatim from A. Schrijver’s lecture notes.