Lecturer: Thomas Rothvoss Due date: Friday, Nov 4, 2022, 11pm, on GradeScope

Problem Set 5

514 - Networks and Combinatorial Optimization

Autumn 2022

Exercise 8.8 (10pts)
Let A be a totally unimodular matrix. Show that the columns of A can be split into two classes such that the sum of the columns in one class minus the sum of the columns in the other class, gives a vector with entries in 0, $+1$ and -1 only.

Exercise (modified from Schrijver – 10pts)
Let A be a totally unimodular matrix and let b be an integer vector and consider the polyhedron $P = \{x \in \mathbb{R}^n \mid Ax \leq b,\ x \geq \mathbf{0}\}$. Prove that for each $y \in (kP) \cap \mathbb{Z}^n$ with $k \in \mathbb{Z}_{\geq 1}$, there are $x^1, \ldots, x^k \in P \cap \mathbb{Z}^n$ so that $y = x^1 + \ldots + x^k$.
Hint. Prove this by induction over k.

Exercise 8.10 (slightly modified; 10pts)
Give a min-max relation for the maximum weight of a stable set in a bipartite graph $G = (V, E)$ without isolated vertices.
Comment. What is meant is that you are given a bipartite graph $G = (V, E)$ and a non-negative integer weight function $w : V \to \mathbb{Z}_{\geq 0}$ and you are asked to find an expression of the form $\min\{...\}$ that equals the maximum of $\sum_{i \in S} w_i$ over all stable sets $S \subseteq V$.

Remark. All three exercises are taken from A. Schrijver's lecture notes where the middle one is somewhat modfied.