Lecturer: Thomas Rothvoss

Due date: Friday, Mar 6, 2026, on GradeScope

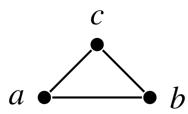
Problem Set 7

409 - Discrete Optimization

Winter 2026

Exercise 1 (10 points)

a) Consider the triangle graph $G = (V,E)$ with 3 nodes and 3 edges



and the matching polytope $P_M = \{x \in \mathbb{R}^E \mid \sum_{e\in\delta(v)} x_e \leq 1 \ \forall v \in V;\ x_e \geq 0 \ \forall e \in E\}$ associated with it. Write it in form $P_M = \{x \in \mathbb{R}^E \mid Ax \leq b\}$ and give the 6×3 constraint matrix A. What 3×3 submatrix A_I of A has $\det(A_I) \notin \{-1,0,1\}$? What is $\det(A_I)$? Compute A_I^{-1}. Which is the extreme point $x = A_I^{-1} b_I$ that belongs to this submatrix?

b) Which of the following matrices is TU? Argue why or why not!

$$A_1 = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -1 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \end{pmatrix}.$$

Exercise 2 (10 points)

Let $G = (V,E)$ be a bipartite graph with parts $V = V_1 \dot\cup V_2$. Consider the linear program:

$$\begin{aligned} \min \sum_{u\in V} y_u & & & \\ y_u + y_v &\geq 1 & & \forall \{u,v\} \in E \\ y_u &\geq 0 & & \forall u \in V \end{aligned}$$

a) If you write the problem in the matrix form $\min\{\mathbf{1}^T y \mid Ay \geq \mathbf{1}; y \geq \mathbf{0}\}$, how is the matrix A defined?

b) Prove that all extreme points of $P = \{y \in \mathbb{R}^V \mid Ay \geq \mathbf{1}; y \geq \mathbf{0}\}$ are integral (A defined as in a)).

c) Which problem that you know from the lecture, does the above LP solve?