Lecturer: Thomas Rothvoss Due date: Friday, Feb 27, 2026, on GradeScope

Problem Set 6

409 - Discrete Optimization

Winter 2026

Exercise 1 (10pts)

(a) Consider $P := \{x \in \mathbb{R}^n \mid Ax = b, x \geq \mathbf{0}\}$ for $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$. Fix a point $y \in P$ that minimizes $|\text{supp}(y)|$ where $\text{supp}(y) := \{j \in \{1, \dots, n\} \mid y_j \neq 0\}$. Prove that $|\text{supp}(y)| \leq m$.
Hint. It might be fruitful to review the proof of Lemma 37 from the lecture notes for this problem.

(b) Let $X \subseteq \mathbb{R}^n$. Prove that for any $x \in \text{conv}(X)$, there is a subset $X' \subseteq X$ with $|X'| \leq n+1$ so that $x \in \text{conv}(X')$.
Hint. Use (a).

Exercise 2 (10pts)
Let $A \in \mathbb{R}^{m \times n}$ and let $b \in \mathbb{R}^m$ with $m \geq n+1$. Suppose that $Ax \leq b$ has no solution x. Prove that there are indices $i_0, \dots, i_n$ so that the system $A_{i_0}^T x \leq b_{i_0}, \dots, A_{i_n}^T x \leq b_{i_n}$ has no solution x.
Hint. You may use a variant of the Farkas Lemma (without proving it) which says that

$$\big(\exists x : Ax \leq b\big) \quad \dot{\vee} \quad \big(\exists y \geq \mathbf{0} : y^T A = \mathbf{0},\ y^T b = -1\big)$$

You may also want to use (a) from the previous exercise.