Lecturer: Thomas Rothvoss Due date: Friday, Mar 7, 2025, in class

Problem Set 7

409 - Discrete Optimization

Winter 2025

Exercise 1 (10 points)
Let $G = (V, E)$ be a bipartite graph with parts $V = V_1 \dot\cup V_2$. Consider the linear program:

$$\begin{aligned} \min \sum_{u \in V} y_u & & \\ y_u + y_v &\geq 1 \quad \forall \{u, v\} \in E \\ y_u &\geq 0 \quad \forall u \in V \end{aligned}$$

a) If you write the problem in the matrix form $\min\{\mathbf{1}^T y \mid Ay \geq \mathbf{1}; y \geq \mathbf{0}\}$, how is the matrix A defined?

b) Prove that all extreme points of $P = \{y \in \mathbb{R}^V \mid Ay \geq \mathbf{1}; y \geq \mathbf{0}\}$ are integral (A defined as in a)).

c) Which problem that you know from the lecture, does the above LP solve?

Exercise 2 (10 points)
Run the Branch & Bound algorithm to solve the following integer linear program

$$\begin{aligned} \max\ 3x_1 + x_2 & & \\ -2x_1 + 3x_2 &\leq 6 \\ 10x_1 + 4x_2 &\leq 27 \\ x_1 &\geq 0 \\ x_2 &\geq \tfrac{1}{2} \\ x_1, x_2 &\in \mathbb{Z} \end{aligned}$$

Also give the Branch & Bound tree.
Hint. For convinience, the underlying polytope looks as follows: