Lecturer: Thomas Rothvoss Due date: Friday, Feb 28, 2025, on GradeScope

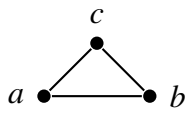
Problem Set 6

409 - Discrete Optimization

Winter 2025

Exercise 1 (10 points)

a) Consider the triangle graph $G = (V, E)$ with 3 nodes and 3 edges



and the matching polytope $P_M = \{x \in \mathbb{R}^E \mid \sum_{e \in \delta(v)} x_e \leq 1 \; \forall v \in V;\; x_e \geq 0 \; \forall e \in E\}$ associated with it. Write it in form $P_M = \{x \in \mathbb{R}^E \mid Ax \leq b\}$ and give the 6×3 constraint matrix A. What 3×3 submatrix A_I of A has $\det(A_I) \notin \{-1, 0, 1\}$? What is $\det(A_I)$? Compute A_I^{-1}. Which is the extreme point $x = A_I^{-1} b_I$ that belongs to this submatrix?

b) Which of the following matrices is TU? Argue why or why not!

$$A_1 = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -1 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \end{pmatrix}.$$