Lecturer: Thomas Rothvoss Due date: Friday, May 20, 2022, on GradeScope

Problem Set 6

409 - Discrete Optimization

Spring 2022

Exercise 1 (10 points)
Let $G=(V,E)$ be a bipartite graph with parts $V=V_1\dot{\cup}V_2$. Consider the linear program:

$$\begin{aligned} \min \sum_{u\in V} y_u & & \\ y_u+y_v &\geq 1 & \forall\{u,v\}\in E \\ y_u &\geq 0 & \forall u\in V \end{aligned}$$

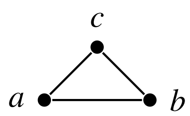
a) If you write the problem in the matrix form $\min\{\mathbf{1}^T y \mid Ay\geq \mathbf{1}; y\geq \mathbf{0}\}$, how is the matrix A defined?

b) Prove that all extreme points of $P=\{y\in\mathbb{R}^V \mid Ay\geq \mathbf{1}; y\geq \mathbf{0}\}$ are integral (A defined as in a)).

c) Which problem that you know from the lecture, does the above LP solve?

Exercise 2 (10 points)

a) Consider the triangle graph $G=(V,E)$ with 3 nodes and 3 edges



and the matching polytope $P_M=\{x\in\mathbb{R}^E \mid \sum_{e\in\delta(v)} x_e\leq 1\ \forall v\in V;\ x_e\geq 0\ \forall e\in E\}$ associated with it. Write it in form $P_M=\{x\in\mathbb{R}^E \mid Ax\leq b\}$ and give the 6×3 constraint matrix A. What 3×3 submatrix A_I of A has $\det(A_I)\notin\{-1,0,1\}$? What is $\det(A_I)$? Compute A_I^{-1}. Which is the extreme point $x=A_I^{-1}b_I$ that belongs to this submatrix?

b) Which of the following matrices is TU? Argue why or why not!

$$A_1=\begin{pmatrix} 1 & 0 & 1 & 0 & 1\\ 0 & 1 & 1 & 1 & 0\\ 0 & 0 & 0 & 1 & 1\\ 1 & 1 & 0 & 0 & 0 \end{pmatrix},\quad A_2=\begin{pmatrix} -1 & 0 & 1 & 0 & -1 & 0 & 0\\ 0 & 1 & 0 & 1 & 1 & 0 & 1\\ -1 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 1 & 1\\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \end{pmatrix}.$$